DOCUMENT RESUME

ED 108 975 SE 019 480

TITLE Program of Studies, Mathematics.
INSTITUTION Fairfax County Schools, Va.
PUB DATE 3 Sep 74
NOTE 219p.

EDRS PRICE MF-$0.76 HC-$10.78 PLUS POSTAGE
DESCRIPTORS Course Content; Curriculum; *Curriculum Guides; *Elementary School Mathematics; Elementary Secondary Education; Mathematics Education; *Objectives; *Program Descriptions; *Secondary School Mathematics
IDENTIFIERS Fairfax County Schools; Virginia

ABSTRACT

This mathematics curriculum guide of the Fairfax County (Virginia) public schools provides a description of the total program, as well as detailed statements related to objectives of the elementary, intermediate, and high school programs and discussions of individual courses. For the elementary grades (K-6), learning objectives are organized by strands, with objectives written for each level within the strand. The levels are designed to promote continuity in each student's progression in the program. The intermediate program, required of all students, provides a bridge to later courses and exposure to a variety of mathematical concepts. The high school program includes 17 courses from which the student may select sequences defined in this document. Objectives are defined for each course. (SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ERIC Full Text Provided by ERIC

CONTENTS

Page

Section A

Program Description 1

Section B

Elementary

Learning Objectives. 1
Elementary Objectives. 1
Scope and Sequence (K-6) 2
Numeration 9
Operations 19
Equations and Inequalities 31
Graphs, Statistics and Probability 35
Geometry 38
Measurement. 44
Money. 51
Time 54

Intermediate

Learning Objectives. 57
Secondary Objectives 57
Sequences of Courses 58
Advanced Placement Sequences 59
Career Mathematics 59
General Mathematics Grade 7. 61
General Mathematics I, Grade 8 73

High School

Learning Objectives. 83
Secondary Objectives 83
Sequences of Courses 84
Advanced Placement Sequences 85
Career Mathematics 85
General Mathematics II, Grade 9. 87
Algebra I, Part I. 95
Algebra I, Part II 97
Algebra I. 99
Geometry 109
Algebra II 119
Algebra II-Trigonometry. 129
Trigonometry 131
Analytic Geometry. 137
Elementary Mathematical Functions. 145
Calculus AB. 151
Calculus BC. 159
Computer Science I and II. 167
Probability and Statistics 185
Senior Mathematics 193
Consumer Mathematics 201

Program Description (K-12)

The mathematics program is designed to develop in students the basic mathematical competencies for

- relating mathematics to their environment
- selecting and using appropriate aids to computation
- participating in today's society
- using applicable mathematical principles in coping with an unknown future
- continuing in education beyond high school

Throughout the program students develop skills and learn concepts. Skills include mastery of facts; concepts include understanding why the facts hold. Skills and concepts may be learned sequentially or concurrently. Together, they enable students to formulate and solve problems in a variety of ways. Individualization, in which each student works at his or her level of ability, with appropriate materials, and at his or her own rate, is a recognized goal of instruction in mathematics.

Elementary Mathematics Program (K-6)

Participation:

Mathematics is required for all students at all levels.

Content:

The content is selected to enable students to develop essential skills. Minimum proficiency levels provide guidance in determining the student's progress throughout the program. By the completion of the elementary school, the student will have

had experiences with

- whole numbers, fractional and decimal numbers, and integers
- ratios, proportions and percents
- simple linear equations and inequalities
- sets of data, statistical averages (mean, median and mode) and graphs
- geometric plane and solid figures such as triangles, squares, rectangles, circles, prisms and cylinders
- United States and metric units of measurement

Students are expected to achieve according to their ability.

Intermediate Mathematics Program (7 & 8)

Participation:

Mathematics is required for all students in grades 7 and 8.

Content:

The content in both years is designed to help students become responsible individuals. The program also provides the beginning courses in a sequence of courses having course credit for graduation from high school. Algebra I is offered to selected students. A few students complete both Algebra I and its succeeding course, geometry.

By the completion of the intermediate school, the student will have had experiences with

- exponents and square roots
- scientific notation
- linear equations and inequalities and related graphs
- elementary probability

- intuitive geometry including concepts of congruency and similarity
- measurement and error
- practical application

Students are expected to achieve according to their ability.

High School Mathematics Program (9-12)

Participation:

Two credits in mathematics, one of which may be grade 8 mathematics, are required for graduation.

Content:

The content depends upon the student's selection of courses from a wide variety of offerings. Sequential courses have prerequisites. A student may select from college preparatory, career, social, or vocational courses. Academic mathematics includes abstract mathematics (structure and logical processes) along with attention to basic skills and practical applications; general mathematics emphasizes basic skills and practical applications. By the completion of high school, the student will have had experiences with

- use of tables and graphs
- formulas, equations and inequalities
- statistics and probability
- measurement and error
- consumer applications
- career related mathematics

Students are expected to achieve according to their ability.

Learning Objectives

The learning objectives are provided as a resource for teachers to use in planning and organizing their program.

The learning objectives are not all inclusive and may be supplemented or adapted by teachers as a basis for establishing the objectives of their program. If an objective is stated for one level, it is usually not repeated in a succeeding level. Thus, objectives in preceding and succeeding levels are vital to the objectives in each level.

Elementary Objectives

The objectives for levels K-6 are organized by strands: Numeration; Operations; Equations and Inequalities; Graphs, Statistics and Probability; Geometry; Measurement; Money; and Time. For each strand, the objectives are written in successive levels. Preceding the objectives is a scope and sequence chart which lists the broad objectives written for each level. The levels are to be viewed as supportive of continuous progress and not as a definitive amount of material to be mastered within a given period of time. The chart may be used in planning and in determining where a student is within a program.

LEVEL K

CONTENT				
Numeration	Shows that the number of objects in two or more sets have certain relationships	Recognizes numerals for cardinal numbers and makes correct assignment of numerals to sets of objects (0 through 10)		
Operations			Begins addition and subtraction	
Equations and Inequalities			Begins to express equalities and inequalities	
Graphs, Statistics & Probability			Begins graphing	
Geometry	Recognizes simple geometric objects in the environment			Begins to describe geometric relationships
Measurement				Begins measurement
Money				Identifies coins
Time				Recognizes that certain parts of the day can be identified Becomes aware that time is measured

LEVEL 1

CONTENT				
Numeration	Expresses whole number numerals and words (0 through 10)	Expresses whole number numerals (0 through 25)	Expresses whole number numerals (0 through 100) Expresses whole numbers in a specified pattern	Recognizes the existence of fractional and negative numbers
Operations	Begins adding (sums through 10) Begins subtracting minuends through 10)	Acquires basic addition skills (sums through 10) Acquires basic subtraction skills (minuends through 10)		Begins adding (sums 11 through 18) Begins multiplying (products through 18) Begins dividing dividends through 18) Begins finding fractional parts of numbers (simple fractions)
Equations and Inequalities			Makes simple statement of equality (addition and subtraction)	Solves selected story problems involving addition or subtraction using objects (whole numbers 1 through 10)
Graphs, Statistics & Probability	Shows that the number line can be used to picture whole numbers			
Geometry			Begins to describe relationships of lines Begins to describe relationships in polygons and circles	
Measurement		Begins measuring familiar objects		
Money				Determines money values
Time				Tells time to the hour and half hour Names consecutively the days of the week and states that there are seven days in a week Names consecutively the months of the year and states that there are 12 months in a year

LEVEL 2

CONTENT				
Numeration	Expresses whole numbers numerals (0 through 1,000)	Arranges whole numbers in a specified pattern		Expresses simple fractions Expresses numbers as Roman numerals (1 through XII)
Operations	Acquires basic addition skills (sums 11 through 18) Reinforces basic subtraction skills (minuends through 10)	Applies addition skills to problems involving up to three addends Acquires basic subtraction skills (minuends through 18)	Applies subtraction skills to problems involving up to three-digit minuends Illustrates the need for negative numbers when trying to subtract a larger number from a smaller number	Acquires basic multiplication skills (products through 25 including 0) Divides with whole number divisors (dividends through 18, no remainder) Begins finding fractional parts of numbers using objects or pictures
Equations and Inequalities			Solves simple equations involving addition or subtraction Solves story problems using simple addition and subtraction facts	Makes simple statements of equality (multiplication and division) Solves simple equations involving multiplication and division
Graphs, Statistics & Probability			Makes a picture of data	
Geometry			Describes relationships of lengths on lines Describes relationships in triangles and between rectangles and squares Begins to describe relationships in solids Begins to describe symmetry	
Measurement		Measures familiar objects		
Money				Determines money values through 99¢
Time				Tells time to the minute

LEVEL 3

CONTENT				
Numeration	Expresses whole number numerals (0 through 1,000,000)	Arranges numbers in multiplication and division patterns	Expresses selected proper fractions Distinguishes between positive and negative numbers (integers)	Continues patterns for simple number sequences
Operations	Demonstrates the commutative and associative properties of addition Applies addition skills to problems involving at least four, two-digit addends Applies subtraction skills to problems involving up to four-digit minuends	Recognizes the properties of multiplication of whole numbers Acquires basic multiplication skills (products through 81) Applies multiplication skills to problems involving up to two-digit multipliers and up to four-digit multiplicands Acquires basic division skills, whole number dividends and divisors (1 through 9)	Begins adding two fractional numbers (unit fractions)	
Equations and Inequalities	Solves simple equations involving addition and subtraction (with grouping symbols)	Solves simple equations involving multiplication or division	Solves selected story problems using simple multiplication and division facts	
Graphs, Statistics & Probability			Shows that graphs can picture numbers and operations	Interprets data Begins to interpret the meaning of a probability experience with two outcomes
Geometry				Describes relationships among line segments, angles and polygons Describes relationships in circles Describes symmetry
Measurement			Measures whole number lengths Measures whole number weights Begins measuring to the nearest fractional part of a unit Begins measuring areas Begins combining units of measurement Creates and solves word problems involving measurements learned to this level	
Money				Determines money values through $10.00 Identifies money values greater than $10.00
Time				Classifies periods of time Solves problems involving time

LEVEL 4

CONTENT				
Numeration	Expresses whole number numerals having up to ten digits Recognizes other symbols and systems for expressing numbers		Expresses proper and improper fractions and mixed numerals Begins decimals (tenths)	Expresses numbers as products of factors
Operations	Recognizes the 0 property of addition Applies addition skills to problems involving at least four addends Applies subtraction skills to problems involving minuends with at least four digits	Recognizes the properties of 0 and 1 in multiplication Applies multiplication skills to problems involving up to three-digit factors Recognizes the properties of division of whole numbers Applies division skills to problems involving multiples of 10 (through 90) as divisors and up to three-digit dividends	Acquires skills for adding and subtracting with fractions and mixed numerals	
Equations and Inequalities	Solves simple equations involving addition and subtraction	Solves simple equations involving multiplication and division	Solves selected word problems requiring a maximum of two different operations	
Graphs, Statistics & Probability			Recognizes that a number line can be a device for picture fractions	
Geometry				Describes relationships of lines and circles Describes relationships in parallelograms Describes relationships in prisms Describes lines of symmetry Begins to describe reflections and rotations
Measurement			Measures length (including some fractional parts) Measures weight (including some fractional parts) Combines units of measurements Identifies liquid measures (whole number units) Creates and solves word problems involving measurements learned to this level	Measures area and perimeter Begins measuring volume
Money				Solves problems involving money values
Time				Classifies time zones Solves problems involving time

LEVEL 5

CONTENT				
Numeration	Expresses whole number numerals having up to 13 digits Expresses numbers in other systems	Expresses prime and composite numbers Expresses fractions in lowest terms	Expresses decimals through thousandths Expresses fractions as decimals Expresses the comparisons between two numbers	Expresses the relationships between metric prefixes and place value
Operations	Applies addition skills to whole numbers Applies subtraction skills to whole numbers Applies multiplication skills to problems involving numbers with at least four digits Applies division skills to problems in which the divisor has up to two digits	Acquires skills for adding with fractions Acquires skills for subtracting with fractions Begins multiplying fractional numbers	Acquires skills for adding with decimals Acquires skills for subtracting with decimals	
Equations and Inequalities	Illustrates the properties of addition and multiplication with equations Solves equations which illustrate inverse operations Solves selected word problems requiring more than one operation			
Graphs, Statistics & Probability	Interprets graphs in the first quadrant of the coordinate plane	Begins to interpret the meaning of chance events Describes data presented graphically Computes the average of a given set of numbers		
Geometry				Begins to describe relationships in three-dimensional space Describes congruent relationships Describes relationships among certain polygons
Measurement				Expresses the relationship among units of weight and among units of length in the metric system Combines units of measurements Begins to use formulas to measure perimeter, area and volume Begins to use ratios in measurement Determines measures of angles Solves word problems involving measurement facts learned to this level
Money			Performs operations using one money value	
Time			Determines time and time periods Solves word problems involving time and requiring a combination of operations	

LEVEL 6

CONTENT eration			
	Expresses whole number numerals in exponential Describes sets using set notation Expresses integer numerals	Expresses complex fractions	Expresses decimals through millionths Expresses relationships between fractions and decimals Expresses ratios Expresses percents
Operations	Applies addition skills to whole numbers Applies subtraction skills to whole numbers Begins adding and subtracting integers Applies multiplication skills to whole numbers Applies division skills to problems in which the divisor has three or more digits	Acquires skills for adding with mixed numerals Acquires skills for subtracting with mixed numerals Acquires skills for multiplying fractions Acquires skills for dividing fractions Acquires skills for multiplying with mixed numerals Acquires skills for dividing with mixed numerals	Applies addition skills to decimals Acquires skills for subtracting with decimals Acquires skills for multiplying with decimals Acquires skills for dividing with decimals Solves problems which require naming a fraction as a decimal or a decimal as a fraction Solves proportions Solves percent problems Solves word problems involving proportions and involving percents
Equations and Inequalities	Solves mathematical sentences with at least three steps	Solves selected word problems	
Graphs, Statistics & Probability	Interprets graphs in the coordinate plane	Determines the probability of an event Interprets circle graphs	
Geometry			Describes certain relationships in triangles, parallelograms and trapezoids Describes relationships in and among solids Describes more than one line of symmetry Describes symmetry in solids
Measurement			Expresses the relationship among units of capacity Combines units of measurement Uses formulas to measure perimeter, area and volume Uses ratios in measurement Solves word problems involving measurement facts learned to this level including fractional parts
Money		Performs operations using two money values	
Time		Determines time and time periods	

NUMERATION

Level K

1. Shows that the number of objects in two or more sets have certain relationships by

   a. Distinguishing between things that are members of a set and things that are not members when given a verbal description of the set

   b. Demonstrating that several sets of objects have a number relationship by

      (1) Making one-to-one matching between members of two equivalent sets

      (2) Distinguishing between pairs of sets whose members can be matched one-to-one and whose members cannot be matched one-to-one

      (3) Identifying sets having no members as the empty set

   c. Identifying which set has more members and which set has fewer members when given two nonequivalent sets

   d. Using correctly the words "more than", "less than", "greatest", "least" and "same as" when given two or three sets of objects

   e. Arranging sets of objects containing 1 through 9 members in one more or one less order

   f. Counting the number of objects in a set by assigning each object the appropriate number

   g. Making patterns using objects one of a kind, two of a kind, and three of a kind in a row, in a column, or on a diagonal; beginning from left to right; or from right to left, or from the middle

   h. Interpreting words such as "top", "bottom", "left", "right", "between", "up", "down", "farther", "farthest", "nearer", "nearest", in activities such as making patterns, placing objects, and following directions

2. Recognizes numerals for cardinal numbers and makes correct assignment of numerals to sets of objects (0 through 10) by

   a. Assigning verbally the appropriate cardinal number to sets containing 1 through 9 objects

   b. Reading orally the numerals in any order

   c. Counting in order from 1 through 9

Numeration

Level K (Cont'd.)

d. Reading the numerals 1 through 9 in sequence

e. Using the symbol 0 to name the number of objects in the empty set

f. Associating the number 10 with a set containing one more object than a set containing 9 objects

Level 1

1. Expresses whole number numerals and knows number words (0 through 10) by

a. Reading orally numerals in any order

b. Constructing a set of a given cardinality

c. Matching numerals with corresponding points on the number line

d. Writing numerals

e. Writing the numerals in order when given ordered sets or pictures of objects

f. Writing the whole number that comes before (one less than) or after (one more than) a given whole number, or between two whole numbers (from 1 through 9)

g. Reading number words from zero through ten and matching the words with an appropriate numeral or set

h. Writing number words when given the numerals in any order as well as in sequential order

i. Using correctly the phrases "more than" and "less than" when comparing the number of objects in two sets

2. Expresses whole number numerals (0 through 25) by

a. Counting from 10 through 19, using 11 as one ten and one, 12 as one ten and two, and so on

b. Counting from 20 through 25, using 21 as two tens and one, 22 as two tens and two, and so on

c. Writing numerals at appropriate points on the number line

d. Selecting and naming the number in a set of objects

e. Writing the digit which is in the tens or ones place for a given numeral

f. Writing the numeral that comes immediately before or after a given whole number, or between two whole numbers (1 through 24)

g. Using correctly one of the symbols $<$, $=$, $>$ between two given numbers

Numeration

Level 1 (Cont'd.)

3. Expresses whole number numerals (0 through 100) by

   a. Grouping by tens and ones

   b. Reading and writing the numerals for sets structured into tens and ones

   c. Reading and writing numerals in sequential order

   d. Counting orally by 1's through 100 in sequence and in short sequences

   e. Writing the numeral that comes immediately before or after a given whole number or between two given whole numbers

   f. Relating place value to groups of tens and groups of ones

   g. Using correctly one of the symbols $<$, $=$, $>$ between two given whole numbers

   h. Writing numerals in expanded notation as ___tens + ___(ones)

4. Expresses whole numbers in a specified pattern by

   a. Naming ordinal position from first through tenth when given an ordered set of objects from 1 through 10

   b. Skip-counting by 5's and 10's through 100

5. Recognizes the existence of fractional and negative numbers by

   a. Reading and writing the fractions $\frac{1}{2}$ and $\frac{1}{4}$ and associating them with appropriate objects or pictures of objects

   b. Illustrating that there are numbers less than zero by using a device such as a thermometer or walk-on number line

Level 2

1. Expresses whole number numerals (0 through 1,000) by

   a. Grouping by hundreds, tens and ones (1 through 99)

   b. Adding 1 to 999 to make one thousand or ten hundreds

   c. Reading and writing numerals (1 through 1,000)

   d. Reading and writing number words for any numeral (0 through 1,000)

   e. Writing the numeral which is one more or one less than a given whole number or between two given whole numbers

   f. Writing a given numeral in expanded notation as ___hundreds + ___tens + ___(ones) (1 through 999)

   g. Completing a place value chart for three-digit numerals

   h. Identifying the place value of each digit for any given whole number (0 through 999)

Numeration

Level 2 (Cont'd.)

i. Using correctly one of the symbols $=$, $\neq$ between two given whole numbers (0 through 1,000)

j. Using correctly one of the symbols $<$, $=$, $>$ between two given whole numbers

2. Arranges whole numbers in a specified pattern by

a. Naming ordinal position from first through hundredth

b. Identifying even and odd numbers (1 through 1,000)

c. Skip-counting by 10's, 5's, 4's, 3's and 2's (1 through 100)

d. Counting down from 25 to 1

3. Expresses simple fractions by

a. Reading and writing the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{3}{4}$

b. Associating fractions with shaded regions and parts of a group of objects

c. Matching selected fractions with appropriate points on the number line

4. Expresses numbers as Roman numerals (I through XII) by

a. Reading and writing Roman numerals

b. Converting Roman numerals to Arabic numerals

Level 3

1. Expresses whole number numerals (0 through 1,000,000) by

a. Grouping by thousands, hundreds, tens and ones (1 through 9,999)

b. Adding one to 9,999 to make ten thousand

c. Reading and writing numerals (0 through 10,000)

d. Grouping by hundred thousands, ten thousands, thousands, hundreds, tens and ones (1 through 999,999)

e. Writing a six-digit numeral in expanded notation

f. Completing a place value chart for six-digit numerals

g. Reading and writing numerals having up to seven digits

h. Reading and writing number words for numerals having up to seven digits

i. Using correctly one of the symbols $<$, $=$, $>$ to order two given whole numbers

j. Rounding to nearest tens, hundreds or thousands

2. Arranges numbers in multiplication and division patterns by

a. Skip-counting by 3's, 4's, 5's and 10's from any whole number starting point (1 through 999)

b. Skip-counting down by 2's from 20 to 0

Numeration

Level 3 (Cont'd.)

3. Expresses selected proper fractions by

   a. Reading and writing proper fractions for appropriate shaded regions or parts of a group of objects

   b. Identifying proper fractions with denominators 1 through 10

   c. Reading and writing a given fraction and illustrating its meaning

4. Distinguishes between positive and negative numbers (integers)

5. Continues patterns for simple number sequences

   e.g., arithmetic: 3, 5, 7, 9,...
   geometric: 2, 4, 8, 16,...

Level 4

1. Expresses whole number numerals having up to ten digits by

   a. Grouping by millions, hundred thousands, ten thousands, thousands, hundreds, tens and ones

   b. Writing a nine-digit numeral in expanded notation

   c. Completing a place value chart for nine-digit numerals

   d. Reading and writing numerals

   e. Using correctly one of the symbols $<$, $=$, $>$ to order two given whole numbers

   f. Rounding to the nearest million, hundred thousand or ten thousand

2. Recognizes other symbols and systems for expressing numbers by

   a. Reading and writing Roman numerals (I through C)

   b. Converting selected Roman numerals (I through C) to Arabic numerals and vice versa

3. Expresses proper and improper fractions and mixed numerals by

   a. Associating a fraction with a number pair where the first number of the pair is a whole number and the second is a whole number not zero

   b. Naming the numerator and the denominator of a fraction

   c. Describing the numerator and the denominator of a fraction

      e.g., $\frac{1}{2}$ means 1 part out of 2 parts

Numeration

Level 4 (Cont'd.)

d. Identifying a fraction having zero numerator with no parts of a whole or no objects in a set

e. Matching fractions with appropriate points on the number line

f. Associating a fraction with the name of a rational number

g. Identifying equivalent proper fractions

h. Building sets of equivalent fractions, including the set with zero as the numerator

i. Using correctly one of the symbols $<$, $=$, $>$ between two given fractions

j. Classifying a proper and an improper fraction

k. Reading and writing mixed numerals

l. Converting an improper fraction to a mixed numeral and vice versa

4. Begins decimals (tenths) by

a. Showing the meaning of tenths by using money values

e.g., 10 pennies equivalent to 1 dime

b. Reading a decimal numeral

c. Naming equivalent fractions and decimals

e.g., $2.6 = 2\frac{6}{10}$

5. Expresses numbers as products of factors by

a. Finding a missing factor of a number when given one factor

b. Finding several factors of a given number

c. Deciding if a given whole number is prime or composite

d. Factoring a given set of whole numbers showing

(1) Greatest common factor

(2) Least common multiple

Level 5

1. Expresses whole number numerals having up to 13 digits by

a. Completing a place-value chart for a given number

b. Writing a given numeral in expanded notation

c. Reading and writing numerals

Numeration

Level 5 (Cont'd.)

d. Using correctly one of the symbols $<, =, >$ between two given whole numbers

e. Rounding a given number

2. Expressing numbers in other systems by

a. Reading and writing Roman numerals (I through M)

b. Converting Roman numerals to Arabic numerals and vice versa

3. Expresses prime and composite numbers by

a. Identifying prime and composite numbers

b. Writing a given composite number as a product of prime numbers

c. Finding the prime factorization of a composite number

d. Determining the least common multiple for a given set of whole numbers

e. Determining the greatest common factor of a given set of whole numbers

f. Factoring the sum of two numbers where each addend has a common factor

e.g., $ab + ac = a(b + c)$

4. Expresses fractions in lowest terms by

a. Identifying which fractions of a set of equivalent fractions are in lowest terms

b. Reducing a fraction to lowest terms

c. Naming a fraction having equal numerator and denominator as the number 1

5. Expresses decimals through thousandths by

a. Identifying and naming corresponding points on the number line

b. Completing a place value chart

c. Reading and writing decimal notation

d. Using correctly one of the symbols $<, =, >$ between two decimals

e. Writing decimals as a whole number numeral $+ \frac{?}{10} + \frac{?}{100} + \frac{?}{1,000}$

e.g., $4.125 = 4 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1,000}$

Numeration

Level 5 (Cont'd.)

f. Rounding to the nearest tenth or hundredth

6. Expresses fractions as decimals by

a. Dividing the numerator of a fraction by the denominator

7. Expresses a comparison between two numbers by

a. Comparing the number of objects in one set to the number of objects in another set and naming the comparison a ratio

b. Expressing a ratio as a fraction

e.g., the ratio of 3 to 4 is $\frac{3}{4}$.

c. Writing many names for a given ratio

e.g., four names for $\frac{1}{2}$ are $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ and $\frac{5}{10}$

8. Expresses the relationship between metric prefixes and place value by

a. Showing that

(1) Kilo means 1,000
(2) Hecto means 100
(3) Deca means 10
(4) Deci means one-tenth
(5) Centi means one-hundredth
(6) Milli means one-thousandth

Level 6

1. Expresses whole number numerals in exponential form by

a. Converting powers of 10 to whole numbers and vice versa

b. Writing a numeral expressed in tens or a multiple of ten as the product of a whole number less than 10 and a power of 10

e.g., $6{,}000 = 6 \times 10^3$

c. Writing a given numeral in expanded notation using exponential form (whole number exponents)

e.g., $7{,}692 = (7 \times 10^3) + (6 \times 10^2) + (9 \times 10) + (2 \times 1)$

2. Describes sets using set notation for

a. Set descriptions

b. Universal sets

c. Subsets

Numeration

Level 6 (Cont'd.)

d. The empty set

e. Set complement

f. Union of sets

g. Intersection of sets

3. Expresses integer numerals by

a. Reading and writing

b. Locating points on the number line corresponding to integers

c. Using correctly one of the symbols $<$, $=$, $>$ to order two given integers

4. Expresses complex fractions by

a. Using the symbol $\div$ interchangeably with the bar of a fraction

b. Reading and writing

e.g., $\dfrac{\frac{2}{3}}{\frac{2}{7}}$ and $\dfrac{3}{\frac{1}{7}}$

c. Naming the reciprocal of any positive rational number

5. Expresses decimals through millionths by

a. Completing a place value chart

b. Reading and writing decimal notation

c. Using correctly one of the symbols $<$, $=$, $>$ to order two given decimals

d. Rounding to the nearest tenth, hundredth or thousandth

6. Expresses the relationship between fractions and decimals by

a. Renaming a fraction or a mixed numeral as a decimal

b. Renaming a terminating decimal as a fraction or a mixed numeral

c. Writing selected repeating decimals as fractions

e.g., $.33 = \frac{1}{3}$

7. Expresses ratios by

a. Using the symbol : to write a ratio

e.g., 2:3

Numeration

Level 6 (Cont'd.)

b. Representing pictorially two equivalent ratios

c. Identifying a proportion as an equation involving two equivalent ratios

d. Reading a given proportion

8. Expresses percents by

a. Picturing the meaning of percent by using a hundred square

b. Using the symbol % to denote percent

c. Writing fractions having denominators of tens or hundreds as percent

d. Renaming whole numbers, fractions and decimals as percents and vice versa

OPERATIONS

Level K

1. Begins adding and subtracting by

   a. Joining two separate sets of objects to obtain sums 1 through 10

   b. Separating 10 or fewer objects into two sets to obtain differences 1 through 10

   c. Solving oral problems using objects 1 through 10

Level 1

1. Begins adding (sums through 10) by

   a. Increasing the number of elements in a set to correspond to a given number and vice versa

   b. Writing the number of objects in each of two disjoint sets and then the number of objects in the two sets joined

2. Begins subtracting (minuends through 10) by

   a. Decreasing the number of elements in a set to correspond to a given number and vice versa

   b. Identifying the number of objects in a given set, then the number of objects in a set removed from the given set, and then the number of objects remaining in the given set

3. Acquires basic addition skills (sums through 10) by

   a. Naming the sum using objects, pictures and/or the number line when given addition statements involving symbols

   b. Solving one-step addition word problems using objects and pictures

   c. Creating a story to describe an addition fact

   d. Finding the sum of two addends both horizontally and vertically:

      e.g., $2 + 3 = 5$ and $\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$

   e. Completing a table for addition

   f. Reciting the addition facts

   g. Using the addition property of zero

   h. Illustrating the associative property of addition by using parentheses

Operations

Level 1 (Cont'd.)

i. Creating and solving one-step addition word problems

j. Achieving 90% on a mastery test

4. Acquires basic subtraction skills (minuends through 10) by

a. Naming the difference using objects, pictures or the number line when given subtraction statements involving symbols

b. Solving one-step word problems involving subtraction using objects or the number line

c. Creating a story to describe a subtraction fact

d. Subtracting both horizontally and vertically

e. Reciting the subtraction facts.

f. Using the subtractive property of zero

5. Begins adding (sums 11 through 18) by

a. Writing the number of objects in each of two disjoint sets and then the number of objects when the sets are joined

6. Begins multiplying (products through 18) by

a. Identifying the number when told there are ___ sets of ___ each when given two or more equivalent sets which together have no more than 18 objects

b. Forming rows and columns of objects to make an array when given 18 or fewer objects

e.g., . . .
. . .

7. Begins dividing (dividends through 18) by

a. Separating a set of 18 or fewer objects into equivalent sets and stating the number of objects in each

b. Separating an array of 18 or fewer objects into equivalent sets and stating the number of objects in each

8. Begins finding fractional parts of numbers (simple fractions) by

a. Dividing a real object and a picture of a real object in halves

b. Dividing a set of real objects and a picture of a set of real objects in halves (10 or fewer objects)

c. Dividing a real object and a picture of real objects in thirds

Operations

Level 1 (Cont'd.)

d. Dividing a set of real objects and a picture of a set of real objects in thirds (3, 6 or 9 objects)

e. Identifying the whole, one-half and one-third of an object

f. Identifying the whole, one-half and one-third of a set of 3, 6 or 9 objects

Level 2

1. Acquires basic addition skills (sums 11 through 18) by

a. Naming the sum using objects, pictures or the number line when given addition statements involving symbols

b. Solving one-step addition word problems using objects and pictures

c. Creating a story problem to describe an addition fact

d. Finding the sum of two addends both horizontally and vertically

e. Using an addition table

f. Reciting the addition facts

2. Reinforces basic subtraction skills (minuends through 10) by

a. Finding differences both horizontally and vertically.

3. Applies addition skills to problems involving up to three addends by

a. Using the associative property to find the sum of three addends (sums through 18)

b. Computing sums to 100 of a multiple of 10 and a number from 0 through 9

c. Computing sums through 90 of two multiples of 10

d. Computing sums through 99 of a number with two digits and a number with one or two digits that do not require regrouping

e. Finding the sum of two and three addends (maximum of four digits each) where regrouping is not required

f. Finding the sum of two addends (maximum of two digits each) where regrouping is required

g. Adding both vertically and horizontally

h. Using correctly one of the symbols $<$, $=$, $>$ when comparing a sum with each addend and vice versa

i. Creating and solving word problems involving addition for sums between 10 and 1;000

Operations

Level 2 (Cont'd.)

4. Acquires basic subtraction skills (minuends through 18) by

   a. Finding differences where regrouping is not required

   b. Finding differences where regrouping is required

   c. Checking subtraction by addition

   d. Creating and solving word problems involving subtraction

5. Applies subtraction skills to problems involving up to three-digit minuends by

   a. Subtracting with regrouping required using two-digit numbers

   b. Subtracting with regrouping required using three-digit numbers

   c. Creating and solving word problems involving subtraction

6. Illustrates the need for negative numbers when trying to subtract a larger number from a smaller number

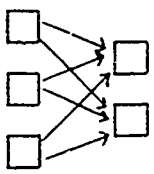
   e.g., $2 - 5 =$ [ ] Expected response cannot be found within set of whole numbers

7. Acquires basic multiplication skills (products through 25, including 0) by

   a. Completing statements such as: five threes are ____; five times three is ____; 5 x 3 = ____ (using objects or the number line)

   b. Completing statements such as: 2 + 2 + 2 = ____; 3 x 2 = ____ (to relate addition and multiplication of whole numbers)

   c. Matching two sets of objects to determine cross products and relating cross products to multiplication

   e.g.,  3 x 2 = 6

   d. Completing a multiplication table

   e. Solving word problems

8. Divides with whole number divisors (dividends through 18, no remainder) by

   a. Completing statements such as (using no more than 18 objects separated into equivalent sets):

      (1) How many objects in all? (6)
      (2) How many in each set? (3)
      (3) How many sets? (2)
      (4) How many threes in 6? (2)

   b. Completing statements such as 6 ÷ 3 = ____

Level 2 (Cont'd.)

9. Begins finding fractional parts of numbers using objects or pictures by
   a. Identifying halves, thirds, fourths and fifths
   b. Dividing a whole into halves, thirds, fourths or fifths
   c. Dividing sets into halves, thirds, fourths or fifths
   d. Identifying two-thirds and three-fourths

Level 3

1. Demonstrates the commutative and associative properties of addition
2. Applies addition skills to problems involving at least four, two-digit addends by
   a. Deciding if the sum is even or odd when given a list of even and odd addends
   b. Determining the sum of four addends, with regrouping required
   c. Finding the sum of at least four addends
   d. Creating and solving word problems requiring addition for sums through six digits
   e. Estimating sums by rounding addends (through six digits)
3. Applies subtraction skills to problems involving up to four-digit minuends by
   a. Finding differences where regrouping is not required
   b. Finding differences where regrouping is required
   c. Estimating differences by rounding to the nearest ten, hundred or thousand
   d. Checking computation using inverse operation
   e. Creating and solving selected word problems
4. Recognizes the properties of multiplication of whole numbers by
   a. Multiplying by 0
   b. Multiplying by 1
   c. Demonstrating the commutative property
   d. Demonstrating the associative property by using parentheses
5. Acquires basic multiplication skills (products through 81) by
   a. Completing a table for multiplication
   b. Achieving 90% on a mastery test

Operations

Level 3 (Cont'd.)

6. Applies multiplication skills to problems involving up to two-digit multipliers and up to four-digit multiplicands by

   a. Determining if a product is odd or even when given a list of even and odd factors

   b. Writing the first nine multiples of each of the numbers 1 through 9

   c. Illustrating that multiplication distributes over addition

      e.g., $2 \times 7 = 2(5 + 2)$

      ..... = ... | ..
      ..... = ... | ..

   d. Multiplying vertically a one-digit factor by a two-digit factor with no regrouping required

   e. Multiplying vertically a one-digit factor by a two-digit factor with regrouping required

   f. Writing and stating orally the products where one factor is a multiple of 10 (10, 20,..., 90) or a multiple of 100 (100, 200, ..., 900)

   g. Multiplying vertically a one-digit factor or a two-digit factor by a two-digit factor (using a short-cut method to denote grouping)

   h. Multiplying a one-digit factor by a three-digit factor or a four-digit factor (both vertically and horizontally)

   i. Estimating products up to two-digit factors by rounding

   j. Finding products where one factor is a whole number (1 through 9) and the other factor is 10

   k. Creating and solving selected word problems

7. Acquires basic division skills, whole number dividends and divisors (1 through 9) by

   a. Relating division of whole numbers to repeated subtraction by using equivalent sets of objects and/or the number line

   b. Dividing where the dividend is the same or less than 81 and the remainder is 0

   c. Achieving 90% on a mastery test for division facts for dividends through 81

   d. Dividing three-digit dividends with last digit 0 by one-digit divisors with remainder 0

   e. Writing and solving a division problem using the symbol ⟌ where the divisor is a one-digit number (writing remainder when necessary)

Operations

Level 3 (Cont'd.)

f. Computing quotients for two- and three-digit dividends and one-digit divisors

g. Checking computation using inverse operation

h. Creating and solving one-step division word problems

8. Begins adding two fractional numbers (unit fractions) by

a Adding two fractions such as $\frac{1}{2}$ and $\frac{1}{2}$ or $\frac{1}{3}$ and $\frac{1}{3}$ using objects or a diagram

Level 4

1. Recognizing the 0 property of addition by

a. Demonstrating the identity property of 0; that is, the sum of a number and 0 is the number

b. Demonstrating that in a subtraction problem when the subtrahend is 0, the difference is the minuend

2. Applies addition skills to problems involving at least four addends by

a. Finding sums for specified addends; adding both vertically and horizontally

b. Estimating sums by rounding

c. Solving selected word problems requiring addition for sums through ten-digit numerals

3. Applies subtraction skills to problems involving minuends with at least four digits by

a. Computing differences
b. Estimating differences by rounding
c. Checking computation using inverse operation
d. Solving selected word problems

4. Recognizes the properties of 0 and 1 in multiplication by

a. Demonstrating the identity property of 1, i.e., the product of a number (other than 0) and 1 is the number

b. Demonstrating the multiplicative property of 0, that is, the product of a number and 0 is 0

5. Applies multiplication skills to problems involving up to three-digit factors by

a. Multiplying using the distributive property

b. Using correctly one of the symbols $<$, $=$, $>$ when comparing factors and products

Operations

Level 4 (Cont'd.)

c. Estimating products by rounding

d. Solving selected word problems

6. Recognizes the properties of division of whole numbers by

a. Recognizing that 0 cannot be a divisor

b. Recognizing that a number divided by 1 is that number

7. Applies division skills to problems involving multiples of 10 (through 90) as divisors and up to three-digit dividends by

a. Dividing a whole number by a one-digit number

b. Dividing using a multiple of (10, 20, ..., 90) as a divisor

c. Dividing a multiple of 100 by a multiple of 10

d. Estimating quotients by rounding

e. Checking computation using inverse operation

f. Creating and solving selected word problems requiring division with one-digit divisors

8. Applies skills for adding and subtracting with fractions and mixed numerals by

a. Adding two proper fractions with like denominators

b. Subtracting two proper fractions with like denominators

c. Checking computation using inverse operation

d. Adding a whole number and a fraction

e. Adding mixed numerals where the mixed numerals have like denominators

f. Solving selected word problems

Level 5

1. Applies addition skills to whole numbers by

a. Estimating sums by rounding to the nearest ten thousand or hundred thousand

b. Solving selected word problems

2. Applies subtraction skills to whole numbers by

a. Estimating differences by rounding to the nearest ten thousand or hundred thousand

b. Subtracting two given numbers

c. Checking computation using inverse operation

d. Solving selected word problems

Operations

Level 5 (Cont'd.)

3. Applies multiplication skills to problems involving numbers with at least four digits by
   a. Estimating products by rounding
   b. Multiplying given factors
   c. Solving selected word problems
4. Applies division skills to problems in which the divisor has up to two digits by
   a. Using a short cut method to divide with one-digit divisors
   b. Dividing using multiples of 10 as divisors
   c. Dividing with two-digit divisors
   d. Dividing using multiples of 100 and using multiples of 1,000 as divisors
   e. Checking computation by inverse operation
   f. Solving selected word problems
5. Acquires skills for adding with fractions by
   a. Adding with fractions having unlike denominators
   b. Estimating solutions and solving word problems
6. Acquires skills for subtracting with fractions by
   a. Subtracting with fractions having unlike denominators (horizontally and vertically)
   b. Checking computation by inverse operation
   c. Estimating solutions and solving word problems
7. Begins multiplying fractional numbers by
   a. Finding a fractional part of a whole number using a pictorial device
   b. Illustrating multiplication with fractions using intersecting regions or the number line
   c. Finding a fractional part of a whole number using no aids
8. Acquires skills for adding with decimals by
   a. Adding with decimals through thousandths
   b. Estimating solutions and solving selected word problems
9. Acquires skills for subtracting with decimals by
   a. Subtracting with decimals through thousandths
   b. Checking computation by inverse operation
   c. Estimating solutions and solving selected word problems

Operations

Level 6

1. Applies addition skills to whole numbers by
   a. Estimating sums by rounding
   b. Solving selected word problems
2. Applies subtraction skills to whole numbers by
   a. Estimating differences by rounding
   b. Subtracting two given numbers
   c. Checking computation by inverse operation
   d. Solving selected word problems
3. Begins adding and subtracting integers by
   a. Finding sums by using a device such as the number line or a grid
   b. Finding differences by using a device such as the number line or a grid
   c. Checking computation by inverse operation
4. Applies multiplication skills to whole numbers by
   a. Estimating products by rounding
   b. Multiplying given factors
   c. Checking computation by inverse operation
   d. Solving selected word problems
5. Applies division skills to problems in which the divisor has at least three or more digits by
   a. Estimating quotients by rounding
   b. Dividing with three- and four-digit divisors
   c. Checking computation by inverse operation
   d. Solving selected word problems
6. Acquires skills for adding with mixed numerals by
   a. Adding using renaming and regrouping
   b. Estimating solutions and solving selected word problems
7. Acquires skills for subtracting with mixed numerals by
   a. Subtracting using renaming and regrouping
   b. Estimating solutions and solving selected word problems
   c. Checking computation by inverse operation
8. Acquires skills for multiplying with fractions by
   a. Multiplying with proper and improper fractions
   b. Estimating solutions and solving selected word problems
   c. Checking computation by inverse operation

Operations

Level 6 (Cont'd.)

9. Acquires skills for dividing with fractions by
   a. Simplifying a given complex fraction
   b. Showing that division with fractions is the inverse operation of multiplication
   c. Dividing with proper and improper fractions
   d. Estimating solutions and solving selected word problems
   e. Checking computation by inverse operation
10. Acquires skills for multiplying with mixed numerals by
   a. Multiplying a mixed numeral by a whole number
   b. Multiplying with mixed numerals using renaming and regrouping
   c. Checking computation by inverse operation
11. Acquires skills for dividing with mixed numerals by
   a. Dividing a mixed numeral by a whole number
   b. Dividing with mixed numerals using renaming and regrouping
   c. Checking computation by inverse operation
12. Applies addition skills to decimals by
   a. Adding with decimals through millionths
   b. Estimating solutions and solving selected word problems
13. Acquires skills for subtracting with decimals by
   a. Subtracting with decimals through millionths
   b. Estimating solutions and solving selected word problems
   c. Checking computation by inverse operation
14. Acquires skills for multiplying with decimals by
   a. Multiplying a number named by a decimal by 10, 100 or 1,000
   b. Multiplying with decimals (through thousandths for each factor)
   c. Estimating solutions and solving selected word problems
   d. Checking computation by inverse operation

Operations

Level 6 (Cont'd.)

15. Acquires skills for dividing with decimals by

    a. Dividing a decimal by a whole number divisor

    b. Dividing a decimal by a decimal

    c. Dividing a whole number by a decimal

    d. Estimating solutions and solving selected word problems

    e. Checking computation by inverse operation

16. Solves problems which require naming a fraction as a decimal or a decimal as a fraction

17. Solves proportions by

    a. Determining if two ratios are a proportion

    b. Computing the missing term in a proportion

18. Solves percent problems by

    a. Finding a requested percent of a given whole number

    b. Determining a proportion in which one ratio has a denominator of 100 and computing the missing term of the proportion

19. Solves selected word problems involving proportions and involving percents

EQUATIONS AND INEQUALITIES

Level K

1. Begins to express equalities and inequalities by

   a. Stating if two sets have the same number of objects when given two sets of 10 or fewer objects

   b. Increasing or decreasing the number of objects in one set so that the set is then equivalent to another set (sets of objects 0 through 10)

   c. Deciding which operation to use when given a story problem requiring addition or subtraction

Level 1

1. Makes simple statements of equality (addition and subtraction) by

   a. Matching the symbols + and = with the words "plus" and "equals" respectively, and vice versa; using ___ + ___ = ___ to make true statements (sums through 10)

   b. Solving addition equations (sums through 10)

      e.g., 2 + 3 = ☐

   c. Stating a corresponding story problem when given an addition equation (sums through 10)

   d. Writing an addition equation from a given addition equation to illustrate the commutative property of addition

      e.g., when given 2 + 3 = 5, writes 3 + 2 = 5

   e. Matching the word "minus" with the symbol − and vice versa; using ___ − ___ = ___ to make true statements (minuends through 10)

   f. Completing related addition and subtraction equations which together show that addition and subtraction are inverse operations (sums through 10)

   g. Checking subtraction by addition

   h. Finding the missing addends (sums through 10)

      e.g., ☐ + 5 = 8 and 8 + ☐ = 10

2. Solves selected story problems involving addition or subtraction using objects (whole numbers 1 through 10)

Level 2

1. Solves simple equations involving addition or subtraction by

   a. Naming the parts of an addition sentence as addends and sum

Equations and Inequalities

Level 2 (Cont'd.)

b. Naming the parts of a subtraction sentence as minuend, subtrahend and difference

c. Filling in + or – to complete an equation for addition or subtraction

d. Finding the missing addends

e. Finding the missing terms in a subtraction statement

f. Writing correctly one of the symbols $<$, $=$, $>$ in mixed addition and subtraction expressions

2. Solves story problems using simple addition and subtraction facts

3. Makes simple statements of equality (multiplication and division) by

a. Matching the symbols x and = with the words "times" and "equals" respectively, and vice versa; using ___ x ___ = ___ to make true statements (products through 25)

b. Solving a multiplication equation

e.g., 2 x 3 = □

c. Stating a corresponding story problem when given a multiplication equation (products through 25)

d. Writing a multiplication equation from a given multiplication equation to illustrate the commutative property of multiplication

e.g., when given 2 x 4 = 8, writes 4 x 2 = 8

e. Matching the words "divided by" with the symbol ÷ and vice versa; using ___ ÷ ___ = ___ to make true statements

f. Completing related multiplication and division equations which together show that multiplication and division are inverse operations

g. Checking division by multiplication

h. Finding the missing factor (products through 25)

4. Solves story problems using simple multiplication or division facts

Level 3

1. Solves simple equations involving addition and subtraction (with grouping symbols) by

a. Writing simplest names for expressions with addition and subtraction with grouping symbols

e.g., (6 - 1) + 7 = □

Equations and Inequalities

Level 3 (Cont'd.)

2. Solves simple equations involving multiplication and division by

   a. Naming the parts of a multiplication sentence as factor and product

   b. Naming the parts of a division sentence as divisor, dividend, quotient and remainder

   c. Filling in x or ÷ to complete an equation for multiplication or division

   d. Finding the missing factor (products through 81)

   e. Finding the missing term in a division statement

   f. Writing correctly one of the symbols $<$, $=$, $>$ in mixed multiplication and division expressions

3. Solves selected story problems using simple multiplication and division facts

Level 4

1. Solves simple equations involving addition and subtraction by

   a. Writing correctly one of the symbols $<$, $=$, $>$ between a pair of expressions involving addition and subtraction

   b. Using the associative property to solve for unknown value

      e.g., $(\square + 6) + 4 = 18$

   c. Finding a missing addend among a list of addends and a given sum

      e.g, $48 + 321 + 528 + \square = 1{,}234$

   d. Converting word problems to mathematical sentences

2. Solves simple equations involving multiplication and division by

   a. Writing simplest names for expressions with multiplication and division with regrouping

      e.g., $2 \times (9 \div 3) =$ ___

   b. Writing correctly one of the symbols $<$, $=$, $>$ between a pair of expressions involving multiplication and/or division

   c. Writing simplest names for expressions involving a combination of multiplication and/or division

   d. Converting word problems to mathematical sentences

3. Solves selected word problems requiring a maximum of two different operations

Equations and Inequalities

Level 5

1. Illustrates the properties of addition and multiplication with equations by

   a. Identifying the properties illustrated by a given equation

      e.g., the equation 2 + 3 = 3 + 2 illustrates the commutative property of addition

   b. Writing an equation for a given property

   c. Writing the properties using letters

      e.g., the commutative property of addition can be written as a + b = b + a

2. Solves equations which illustrate inverse operations

   e.g., if x + 2 = 6, then x = 6 - 2 or x = 4

3. Solves selected word problems requiring more than one operation

Level 6

1. Solves mathematical sentences with at least three steps

2. Solves selected word problems

GRAPHS, STATISTICS AND PROBABILITY

Level K

1. Begins graphing by

   a. Making a simple chart using cutouts or objects

   b. Making two-dimensional patterns on a grid such as a geoboard, pegboard or squared paper

Level 1

1. Shows that the number line can be used to picture whole numbers by

   a. Writing numerals at appropriate points

Level 2

1. Makes a picture of data by

   a. Organizing a set of data; i.e., separating a set of approximately 15 different objects into subsets based upon an attribute such as color

   b. Constructing a bar graph

Level 3

1. Shows that graphs can picture numbers and operations by

   a. Locating integers on the number line

   b. Locating whole number points on a grid

   c. Locating integer points on a grid

2. Interprets data by

   a. Reading a table showing tallied data

   b. Organizing experimental data by making tallies and tables

   c. Reading a bar graph

3. Begins to interpret the meaning of a probability experiment with two outcomes by

   a. Stating the possible outcomes in a simple probability experiment and then naming the outcome which is more likely

   b. Determining the chance in a simple probability experiment; i.e., "What is the chance of drawing a red marble from a bag containing one red and five blue marbles?"

Graphs, Statistics and Probability

Level 4

1. Recognizes that the number line can be a device for picturing fractions by

   a. Locating points between 0 and 1

   b. Matching a set of equivalent fractions to a single point

Level 5

1. Interprets graphs in the first quadrant of the coordinate plane by

   a. Locating a given ordered pair of numbers

   b. Completing a table to make a graph and reading a graph to make a table

   c. Graphing a function involving addition or subtraction of whole numbers

   d. Reading a simple line graph and making inferences from the graph

2. Begins to interpret the meaning of chance events by

   a. Giving examples of chance events and examples of certain events

   b. Writing a fraction ($\frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$) to indicate the chance of an event

   c. Determining if two events are equally likely

3. Describes data presented graphically by

   a. Reading and making picture graphs, bar graphs and line graphs

4. Computes the average of a given set of numbers

Level 6

1. Interprets graphs in the coordinate plane by

   a. Drawing the number line depicting the integers

   b. Completing a table to make a graph and reading a graph to make a table

   c. Graphing a function involving addition or subtraction of integers

   d. Recognizing that a table for a graph is a set of ordered pairs

   e. Graphing a function table involving addition or subtraction of nonnegative rational numbers in the coordinate plane

Graphs, Statistics and Probability

Level 6 (Cont'd.)

2. Determining the probability of an event by

   a. Conducting a probability experiment with up to six possible outcomes and recording the result in a table

   b. Conducting a probability experiment with several outcomes where each outcome is an ordered pair and recording the result on a whole number grid

   c. Conducting a probability experiment with a set of equally likely outcomes (up to 16) and recording the outcomes in an array; writing the probability of an outcome as a fraction

3. Interprets circle graphs

GEOMETRY

Level K

1. Recognizes simple geometric objects in the environment by.
   a. Identifying simple closed curves in a set of curves
   b. Selecting and naming orally models of circles, squares, rectangles and triangles
   c. Selecting and naming orally models of cubes and cylinders
2. Begins to describe geometric relationships by
   a. Identifying, continuing or copying repetitive patterns of geometric shapes
   b. Using correctly the terms "inside", "outside" and "on" as related to plane geometric figures

Level 1

1. Begins to describe relationships of lines by
   a. Identifying lines, line segments and curved lines
   b. Identifying the longest and shortest when given models of line segments of different lengths
   c. Making pencil drawings of lines and line segments
   d. Using correctly the terms horizontal, vertical and slanting
   e. Recognizing a grid as a pattern of intersecting horizontal and vertical lines
   f. Recognizing parallel lines
2. Begins to describe relationships in polygons and circles by
   a. Identifying models of circles, rectangles, triangles and squares
   b. Constructing a square, a rectangle, and a triangle using a pegboard or geoboard and a rubber band
   c. Making pencil drawings of circles, squares, rectangles and triangles
   d. Using correctly the terms side and corner
   e. Deciding if a given point is inside, outside or on a simple closed curve
   f. Identifying geometric figures in familiar things (earth, moon, room, etc.)

Geometry

Level 2

1. Describes relationships of lengths on lines by

   a. Constructing line segments of various lengths and comparing lengths (longest, longer, shortest, short or equal) when given a pegboard or a geoboard and a rubber band

   b. Drawing a curve of about the same length as a given curve

   c. Distinguishing between line segments and lines by drawing models of each

2. Describes relationships in triangles and between rectangles and squares by

   a. Constructing triangles of different shapes using straws or sticks

   b. Drawing a recognizable triangle using a straightedge

   c. Drawing a recognizable square and a recognizable rectangle using a straightedge

3. Begins to describe relationships in solids by

   a. Identifying cylinders, prisms, pyramids, cones and spheres

   b. Using correctly the terms edge and vertex

   c. Naming figures formed by the edges of one side of a prism

4. Begins to describe symmetry by

   a. Folding a symmetric object along a line segment so that the two parts have exactly the same shape

   b. Determining if a given figure is symmetric with respect to a given line segment

Level 3

1. Describes relationships among line segments, angles and polygons by

   a. Identifying points in space; labeling a point

   b. Labeling a line and line segment

   c. Drawing and labeling a ray

   d. Drawing and labeling an angle

   e. Comparing sizes of angles by tracing

   f. Constructing a right angle using a pegboard or geoboard and by folding paper

Geometry

Level 3 (Cont'd.)

g. Constructing a square, a rectangle, and a right triangle using a geoboard or a pegboard, a straightedge, and a model of a right angle

h. Constructing a square, a rectangle, and a right triangle using a straight edge and a model of a right angle

i. Identifying right isosceles triangles

j. Counting the number of segments and angles in a given plane figure

k. Identifying quadrilaterals, pentagons and hexagons

l. Identifying regular polygons

m. Drawing and counting the diagonals in a given polygon

n. Constructing parallel lines by devices such as folding paper, straightedges and models of angles

o. Illustrating that two lines perpendicular to the third line are parallel

p. Identifying models of parallelograms and rhombuses

q. Labeling polygons

2. Describes relationships in circles by

a. Folding a circular region to obtain four right angles at the center

b. Identifying the radius and the diameter of a circle

c. Drawing a circle using a model

d. Constructing a circle using a given line segment to obtain equal segments from a given point

3. Describes symmetry by

a. Drawing examples of line symmetry

b. Finding the midpoint of a line segment by folding paper

Level 4

1. Describes relationships of lines and circles by

a. Constructing a circle with a compass

b. Labeling a circle

c. Identifying a tangent to a circle

Level 4 (Cont'd.)

d. Identifying and drawing central and inscribed angles

e. Identifying inscribed and circumscribed circles when given models

2. Describes relationships in parallelograms by

a. Drawing a parallelogram

b. Showing that squares and rectangles are special parallelograms

c. Identifying squares as special kinds of rectangles

3. Describes relationships in prisms by

a. Using correctly the terms face, vertex and surface

b. Counting the vertices and faces of a cube

c. Identifying unseen parts of a regular solid when given sufficient parts that can be seen

d. Drawing a recognizable prism

4. Describes lines of symmetry by

a. Drawing the line of symmetry in a given symmetric figure

b. Completing a figure to make it symmetric

c. Identifying symmetric points on a grid

5. Begins to describe reflections and rotations by

a. Selecting which pairs of figures in a given set of pairs are reflections

b. Identifying a line of reflection in a given reflection

c. Identifying a shape which has been rotated

d. Rotating a shape to fit within a specified position

Level 5

1. Begins to describe relationships in three-dimensional space by

a. Recognizing the difference between intersecting lines and nonintersecting lines

b. Identifying planes

c. Recognizing the relationship among lines and planes in space (including intersecting lines, intersecting lines and planes, and intersecting planes)

Level 5 (Cont'd.)

d. Drawing a recognizable cylinder, cone, pyramid, rectangle, prism and sphere

2. Describes congruent relationships by

a. Constructing congruent line segments with a compass and straight edge

b. Comparing angles by using a compass and a straight edge

c. Identifying acute and obtuse angles

d. Identifying vertical angles and showing that they are congruent

e. Determining congruent parts of a given geometric figure

f. Determining if two figures are congruent by considering corresponding parts

g. Constructing, using a compass and a straight edge, each of the following:

(1) Congruent angles
(2) An angle bisector
(3) A line segment bisector
(4) Congruent circles
(5) Congruent triangles
(6) Perpendicular lines

3. Describes relationships among certain polygons by

a. Identifying acute and obtuse triangles

b. Showing how quadrilaterals, parallelograms, rectangles, squares, trapezoids, and rhombuses are alike and how they are different

c. Identifying octagons

d. Distinguishing between regular and irregular polygons

Level 6

1. Describes certain relationships in triangles, parallelograms and trapezoids

a. Identifying isosceles and equilateral triangles

b. Identifying trapezoids and isosceles trapezoids

c. Determining altitudes and bases of triangles and parallelograms

d. Determining if two triangles are similar by considering corresponding parts

Geometry

Level 6 (Cont'd.)

2. Describes relationships in and among solids by

   a. Identifying the altitude and base of a rectangular prism

   b. Identifying the five regular solids by naming and counting faces

   c. Identifying cross sections of regular solid figures

3. Describes more than one line of symmetry by

   a. Identifying lines of symmetry in figures such as squares, rectangles, triangles and circles

4. Describes symmetry in solids by

   a. Identifying the planes of symmetry

MEASUREMENT

Level K

1. Begins measurement by

   a. Using correctly such terms as:

longer, shorter	highest, lowest
longest, shortest	same size
higher, lower	

   b. Using various objects to measure the length of a given object

   c. Using correctly such temperature terms as:

hot, cold	hotter, colder
warm, cool	freezing, boiling

Level 1

1. Begins measuring familiar objects by

   a. Selecting correct illustrations for such pairs as:

far, near	heavy, light
big, little	right, left
tall, short	more, less

   b. Comparing the lengths of a set of similar concrete objects (such as rods) and deciding how many of one length it takes to make another length

   c. Measuring length using a primary ruler or another suitable object marked off in inches

   d. Measuring length using a primary ruler or another suitable object marked off in centimeters

   e. Ordering objects according to a given property such as volume, area, length and number (through 10 objects)

   f. Showing and stating how many cups it takes to fill a pint; how many pints to fill a quart; how many cups to fill a quart

   g. Identifying one dozen and one-half dozen objects by counting

   h. Recognizing that "pound" is a unit of weight

      e.g., a pound of sugar or, "I weigh ____ pounds"

   i. Recognizing that things can be weighed in grams or kilograms

      e.g., "I weigh ____ kilograms" and "This object weighs ____ grams"

Measurement

Level 2

1. Measures familiar objects by

   a. Drawing a line segment about as long as a given line segment

   b. Giving an example of about how long an inch is and about how long a centimeter is

   c. Determining and drawing lengths in inches and centimeters

   d. Describing the relationship between centimeters and inches

   e. Estimating lengths in inches and centimeters

   f. Showing and stating that 12 inches = 1 foot and 3 feet = 1 yard using a foot ruler and a yardstick

   g. Showing and stating that four quarts fill a gallon and vice versa

   h. Converting from cups to pints to quarts to gallons and vice versa (whole number quantities only)

   i. Identifying an ounce as a unit for measuring weight and weighing to the nearest ounce (through 16 ounces) on a balance scale

   j. Reading a Fahrenheit and Centigrade thermometer and recording temperatures

   k. Solving story problems using measurement facts learned to this level

Level 3

1. Measures whole number lengths by

   a. Comparing two or more standard units of length

      e.g., inch, foot, yard

   b. Converts feet to yards, inches to feet, inches to yards and vice versa

   c. Measuring segments in centimeters and millimeters

   d. Showing the relationship between millimeters and centimeters

      e.g., 3 centimeters = 30 millimeters
      60 millimeters = 6 centimeters

   e. Stating that 100 centimeters = 1 meter; converting centimeters to meters and centimeters; converting meters and centimeters to centimeters

Measurement

Level 3 (Cont'd.)

f. Stating that 1 kilometer = 1,000 meters; converting meters to kilometers and vice versa

e.g., 7,000 meters = 7 kilometers
6 kilometers = 6,000 meters

g. Comparing a meter stick to a yardstick to observe their relative lengths

h. Selecting the appropriate unit for a measurement when given two or more objects of different lengths

i. Finding perimeter (in whole inches, whole centimeters or whole millimeters) of polygons having no more than six sides

j. Illustrating that measurement is an approximation

2. Measures whole number weights by

a. Stating that 16 ounces = 1 pound; identifying the number of ounces in one-half and one-fourth pound

b. Converting pounds and ounces to ounces; converting ounces to pounds and ounces

c. Stating that 1,000 grams = 1 kilogram; converting kilograms to grams and vice versa (whole number quantities only)

e.g., 2 kilograms = 2,000 grams
3,000 grams = 3 kilograms

d. Describing the relationship between grams and ounces and between kilograms and pounds

3. Begins measuring to the nearest fractional part of a unit by

a. Measuring and estimating length to the nearest inch and to the nearest one-half inch

b. Solving problems using gallons and half gallons

e.g., two and one-half gallons is 5 half gallons

4. Begins measuring area by

a. Finding the area of a rectangle by using a square grid

5. Begins combining units of measurement by

a. Finding the difference in two temperature readings (whole number quantities only)

6. Creates and solves word problems involving measurements learned to this level

Measurement

Level 4

1. Measures length (including some fractional parts) by

   a. Stating that 10 meters = 1 decameter, 10 decameters = 1 hectometer, 10 hectometers = 1 kilometer; 10 millimeters = 1 centimeter, 10 centimeters = 1 decimeter; 10 decimeters = 1 meter and shows the relationship among the metric units of length

   b. Using the symbols mm, cm, dm, m, dam, hm and km

   c. Converting using millimeters, centimeters, decimeters and meters

   d. Converting using meters, decameters, hectometers and kilometers

   e. Stating that 5,280 feet = 1 mile and converting miles to feet and vice versa

      e.g., 6,320 feet = 1 mile + 1,040 feet

   f. Describing the relationship between the kilometer and the mile

2. Measures weight (including some fractional parts) by

   a. Converting kilograms to grams and vice versa (using the fractions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{10}$)

      e.g., 2,500 grams = $2\ \frac{1}{2}$ kilograms

   b. Using the symbols kg and g

   c. Stating that 2,000 pounds = 1 ton and converting tons to pounds, and vice versa (whole number and one-half quantities only)

   d. Identifying the number of ounces in $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ of a pound

3. Combines units of measurement by

   a. Adding and subtracting measurements involving feet and inches, using regrouping, and simplifying answers

   b. Adding and subtracting measurements involving pounds and ounces, using regrouping, and simplifying answers

   c. Adding and subtracting measurements involving gallons and quarts, using regrouping, and simplifying answers

   d. Reading a thermometer for degrees above and below zero and telling temperature change

      e.g., at 30° above zero, a drop of 40° is 10° below zero

4. Identifies liquid measures (whole number units) by

   a. Stating that 1,000 liters = 1 kiloliter

   b. Using the symbols l and kl

Measurement

Level 4 (Cont'd.)

c. Converting liters to kiloliters and vice versa

d. Comparing the relative sizes of quarts and liters

5. Creates and solves word problems involving measurements learned to this level

6. Measures area and perimeter by

a. Estimating the area of a given polygon by using a square grid

b. Finding the area of a square or rectangle

c. Illustrating that perimeter is a measure of length

d. Comparing the areas of two squares

e. Finding perimeter of a given polygon

f. Comparing, by subtraction, the perimeters of two given polygons

7. Begins measuring volume by

a. Using cubic units to determine volume of a rectangular prism

b. Using the symbol sq.

Level 5

1. Expresses the relationship among units of weight and among units of length in the metric system by

a. Stating how kilograms, hectograms, decagrams, grams, decigrams, centigrams and milligrams are related to expanded notation

b. Converting each unit of weight to another unit

c. Stating how units of length are related to expanded notation

d. Converting each unit of length to another unit

e. Showing that all measurements are approximate by using smaller units to express a more precise measurement

2. Combines units of measurement by

a. Adding metric units of weight

e.g., 9 grams + 6 milligrams = 900.6 milligrams
or 9.006 grams

b. Adding metric units of length

e.g., 8 meters + 6 centimeters = 806 centimeters
or 8.06 meters

Level 5 (Cont'd.)

3. Begins to use formulas to measure perimeter, area and volume by

   a. Finding the perimeter of regular polygons by multiplying

   b. Showing that $\pi = \frac{\text{circumference}}{\text{diameter}}$ and that π is approximately 3.14 or $3\frac{1}{7}$

   c. Finding the circumference of a circle

   d. Finding the area of a rectangle

   e. Finding the volume of a right rectangular prism

   f. Finding the surface area of a solid by measuring, by experimentation and by formula

   g. Estimating the area of a circle by using a square grid

4. Begins to use ratios in measurement by

   a. Making and reading scale drawings

   b. Using the formula d = rt to solve distance problems

5. Determines measures of angles by

   a. Using a protractor

6. Solves word problems involving measurement facts learned to this level

Level 6

1. Expresses the relationship among units of capacity

   a. Stating how the metric units are related to expanded notation

   b. Converting each metric unit to another metric unit

   c. Describing the relationships between quarts and liters

   d. Stating that 8 fluid ounces = 1 cup; converting among ounces, cups, pints and quarts

2. Combines units of measurement by

   a. Adding metric units of capacity

      e.g., 2 hectoliters + 3 liters = 203 liters or 2.03 hectoliters

   b. Multiplying units of measurement using regrouping and renaming

3. Uses formulas to measure perimeter, area and volume by

   a. Finding the perimeter of a rectangle and the perimeter of a square

Measurement

Level 6 (Cont'd.)

b. Finding the area of a square, a parallelogram and a triangle

c. Finding the area of a polygon composed of different kinds of polygons

d. Finding the volume of a right prism using the formula $V = Bh$

e. Using the symbol cu.

f. Finding the surface area of a solid using the formula for each of its parts

4. Uses ratios in measurements by

a. Making and reading scale drawings where measurements include those with fractional parts

b. Comparing two given lengths

c. Determining a missing side in similar triangles by using a proportion

5. Solves word problems involving measurement facts learned to this level including fractional parts

MONEY

Level K

1. Identifies coins by

   a. Selecting a requested coin when given a penny, nickel, dime or quarter

Level 1

1. Determines money values by

   a. Matching pennies, nickels, and dimes with their numerical values and with values in other coins

      e.g., 10 pennies = 1 dime

   b. Naming the value of a collection of pennies, nickels and dimes using the symbol ¢ (sums to 25¢ and selected greater amounts)

   c. Identifying prices of objects with appropriate sets of coins

   d. Solving story problems about money (sums through 18¢) using objects

   e. Making change for money values by counting up to the greater value

Level 2

1. Determines money values through 99¢ by

   a. Matching a quarter with its numerical value or with its value in other coins

   b. Naming the value of small collections of coins (pennies, nickels, dimes and quarters) using the ¢ symbol and stating equivalent coin combinations (through 99¢)

   c. Solving story problems involving money values

Level 3

1. Determines money values through $10.00 by

   a. Determining which of two collections of coins is greater than, less than or equal to the other

   b. Determining the greatest and the least collection in more than two collections of coins

   c. Matching a dollar with its value in a collection of coins

Money

Level 3 (Cont'd.)

d. Naming the value of a collection of dollars and coins

e. Writing money values using the words "dollar(s)" and "cent(s)" and writing a combination of dollars and cents

e.g., $7.68

2. Identifies money values greater than $10.00 by

a. Finding the total amount in dollars and cents of two or more money amounts

b. Finding the difference in the amounts when given two money amounts

c. Making change for a single purchase by counting up through $1.00 from a purchase price which is less than $1.00

d. Making change for several purchases from selected money amounts

e. Solving one-step word problems involving addition of money values using cent notation and decimal notation

f. Solving one-step word problems involving subtraction of money values using cent notation and decimal notation

Level 4

1. Solves problems involving money values by

a. Making change for several purchases by counting up through a stated amount when the total purchase price is less than $1.00.

b. Adding money values (in horizontal and vertical form)

c. Subtracting money values (in horizontal and vertical form)

d. Solving word problems involving addition of money values

e. Solving word problems involving subtraction of money values

Level 5

1. Performs operations using one money value by

a. Multiplying using whole number multipliers

b. Dividing using whole number divisors

c. Solving word problems

Money

Level 6

1. Performs operations using two money values by

   a. Multiplying using a money value as the multiplicand

   b. Dividing using a money value as the dividend

   c. Solving word problems

TIME

Level K

1. Recognizes that certain parts of the day can be identified by

   a. Using correctly terms such as "morning", "noon", "afternoon", "evening", "night", "earlier" and "later" to respond to questions such as "What part of the day is this?"

2. Becomes aware that time is measured

   e.g., Uses a timing device such as an hourglass to determine if a given task can be done before the sand runs out

Level 1

1. Tells time to the hour and half-hour by

   a. Identifying that a circular region is separated into 12 equal parts for a round clock and writing numerals 1 through 12 on the marks denoting the separation

   b. Determining that the minute hand is longer than the hour hand

   c. Placing correctly the hour hand and the minute hand on a clock face

   d. Stating that it is after ___ o'clock when presented with a clock face which has just the hour hand pointing between any two numerals

   e. Placing the minute hand at the numeral 6 and the hour hand midway between two consecutive numerals to tell time to the nearest half hour

   f. Reading and writing hour times

   g. Reading and writing half-hour times

      e.g., 4:30

2. Names consecutively the days of the week and states that there are seven days in a week

3. Names consecutively the months of the year and states that there are 12 months in a year

Level 2

1. Tells time to the minute by

   a. Identifying that each space between any two consecutive numerals on a clock face is separated into five sections

Level 2 (Cont'd.)

b. Placing an arrow on a clock number line to identify number of minutes after the hour

c. Identifying that the minute hand moves from one numeral to a succeeding one in five minutes

d. Counting marks on a clock number line, responding with the word "minutes" when asked to count the number of marks

e. Writing time to five minutes

e.g., 9:05 and five minutes after nine o'clock

f. Writing time to minutes

e.g., 9:02 and two minutes after nine o'clock

Level 3

1. Classifies periods of time by

a. Stating that 60 minutes = 1 hour

b. Stating that 60 seconds = 1 minute

c. Using a.m. and p.m. to describe parts of the day

d. Stating that there are 24 hours in one day

e. Stating that there are approximately 52 weeks in one year and approximately 365 days in one year

2. Solves problems involving time by

a. Converting between selected time units using hours, minutes and seconds

e.g., $1\frac{1}{2}$ hours = 90 minutes

b. Showing that a certain time may be stated as minutes before an hour and minutes after an hour

c. Writing any given date with numerals for the month, day and year

d. Creating and solving word problems using a calendar or a clock

Level 4

1. Classifies time zones by

a. Naming the time zones in North America

b. Identifying on a map the time zones in North America

Time

Level 4 (Cont'd.)

2. Solves problems involving time by

   a. Finding the number of minutes which have elapsed between two minute hand readings for time differences

   b. Adding or subtracting time units using regrouping

   c. Converting using seconds, hours, days, weeks, months and years (whole numbers only)

Level 5

1. Determines time and time periods by

   a. Telling what time it will be after adding, subtracting or multiplying using hours on a given clock face

   b. Calculating periods of time

      e.g., years, decades, centuries

2. Solves word problems involving time and requiring a combination of operations

Level 6

1. Determines time and time periods by

   a. Converting among hours, minutes, seconds and fractional parts of a day

   b. Telling time using a 24-hour clock

   c. Interpreting time tables

      e.g., bus, train and airline schedules

Learning Objectives

The learning objectives are provided as a resource for teachers to use in planning and organizing their program.

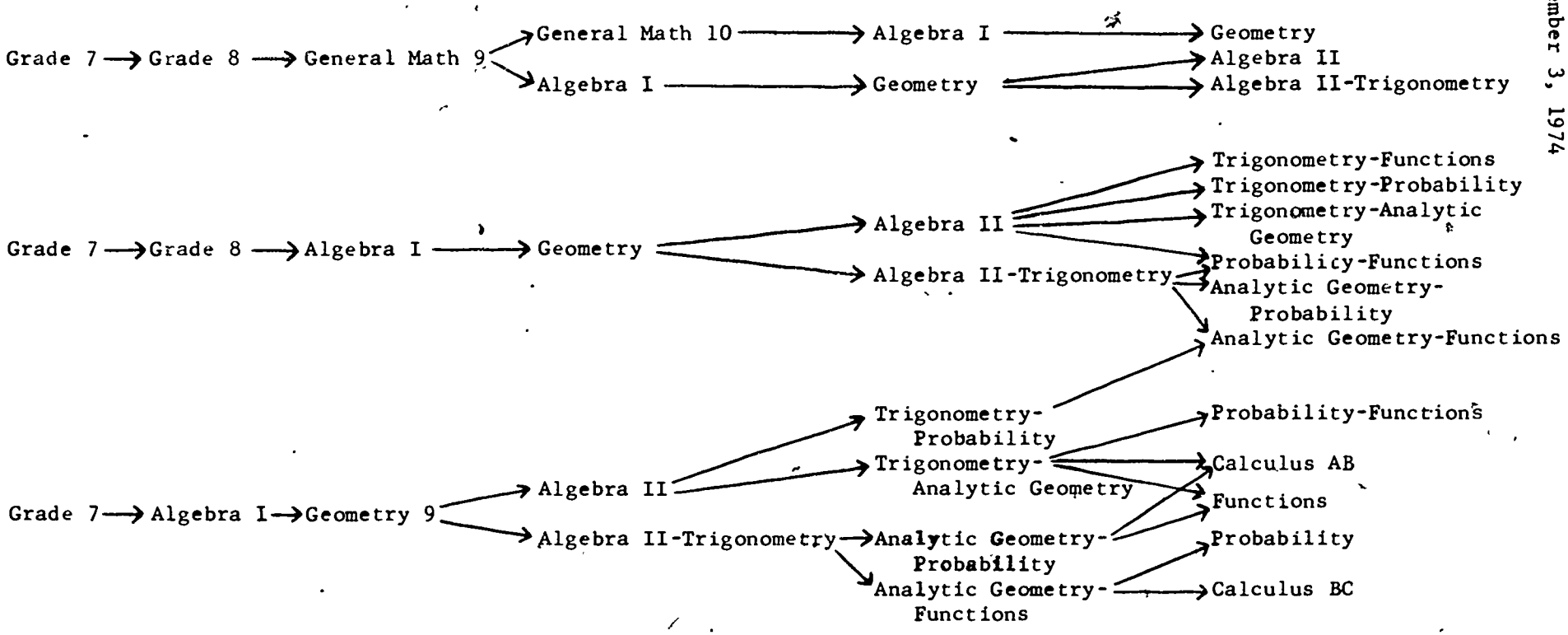
The learning objectives are not all inclusive and may be supplemented or adapted by teachers as a basis for establishing the objectives of their program. If an objective is stated for one level, it is usually not repeated in a succeeding level. Thus, objectives in preceding and succeeding levels are vital to the objectives in each level.

Secondary Objectives

The secondary objectives are organized by courses. For each course, the course name, code, prerequisites, and description are stated. The objectives are written in sequential units. No time limit is suggested for any one unit.

Preceding the course objectives is a sequence chart. The chart may be used in planning an appropriate secondary mathematics program.

SEQUENCES OF COURSES



Computer Science 1 - a one semester course that can be taken after the successful completion of Algebra I.

Computer Science 2 - a one year course that can be taken after or in conjunction with Algebra II or Algebra II-Trigonometry.

Senior Mathematics - a course designed to offer opportunities to concentrate on selected topics.

Consumer Mathematics - a course designed to help the student become a judicious consumer that can be taken by 10th grade students (after successful completion of Geometry) or 11th and 12th grade students.

Advanced Placement Sequences

In mathematics two advanced placement sequences are offered for students who desire to receive college credit for mathematics content that is completed in high school. Those students who take Calculus BC can earn up to one full year of credit, while students who take Calculus AB can earn one semester credit.

Comprehensive examinations for Calculus AB and Calculus BC are administered by the College Entrance Examination Board to determine the amount of college credit that is earned.

Grade 7
↓
Algebra I
↓
Geometry
↓
Algebra II-Trigonometry
↓
Analytic Geometry-Functions
↓
Calculus BC

Grade 7
↓
Algebra I
↓
Geometry
↓
Algebra II
↓
Trigonometry-Analytic Geometry
↓
Calculus AB

Career Mathematics

The academic career related courses beyond geometry are:

Trigonometry
Computer Science I
Computer Science II
Probability and Statistics
Consumer Mathematics

General Mathematics Grade 7

General Mathematics Grade 7 reinforces and extends the mathematics presented in the levels K-6 and shows how it applies in the real world. Students should be given the opportunity of working problems of an extended nature such as

$$\begin{array}{r} 147\ \frac{13}{20} \\ 48\ \frac{3}{8} \\ +\ 53\ \frac{5}{6} \\ \hline \end{array}$$

Upon completion of General Mathematics Grade 7 the student should

1. Operate with facility and accuracy on non-negative rational numbers
2. Recognize relationships existing among familiar geometric figures
3. Apply mathematics to some real life situations

GENERAL MATHEMATICS GRADE 7 (180 days)

Units

A. Number Theory

B. Non-negative Rational Numbers

C. Exponents and Expanded Notation

D. Ratio, Proportion, Percent

E. Geometry

F. Probability

G. Measurement

H. Statistics

I. Integers

J. Variables

K Graphs

GENERAL MATHEMATICS GRADE 7

A. Number Theory

1. Becomes familiar with number patterns used in problem solving by

   a. Identifying patterns for the product of even and odd whole numbers

   b. Expressing consecutive whole numbers, consecutive even numbers, and consecutive odd numbers using symbols

      e.g., $2n$ is even and $2n + 1$ is the next odd

2. Finds factors of given whole numbers by

   a. Identifying the prime numbers less than 100

   b. Using the rules for divisibility by 2, 3, 4, 5, 9, 10

   c. Writing the prime factorization of a given whole number

   d. Finding the greatest common factor of two or more given whole numbers

   e. Determining if two given whole numbers are relatively prime

3. Finds multiples of given whole numbers by

   a. Identifying multiples of a given whole number

   b. Determining if one given whole number is a multiple of another given whole number

   c. Determining the least common multiple for two or more given whole numbers

4. Increasing or decreasing the numerator or the denominator of a given fraction and determining if the resulting fraction is greater than or less than the original fraction

B. Non-negative Rational Numbers

5. Becomes proficient in operating on whole numbers by solving problems such as

   a. $5(85 + 54)$

b. $7(5 + 4) - (8 \div 7)$

c. $\frac{30 \times 92}{920}$

d. $79{,}912 \div 405$

6. Orders non-negative rational numbers by

   a. Graphing on a number line the point corresponding to a given non-negative rational number

   b. Writing correctly the symbols $<, =, >$ to order two given non-negative rational numbers

   c. Finding a number between two given non-negative rational numbers

7. Illustrates that there are different ways of representing a non-negative rational number by

   a. Writing a given mixed numeral as a fraction ($\frac{a}{b}$, where a and b are whole numbers, $b \neq 0$) and vice versa

   b. Writing a given decimal as a fraction and vice versa

   c. Writing a repeating decimal as a fraction and vice versa

   d. Selecting the equivalent fractions from a given list of fractions

   e. Reducing a fraction to lowest terms

8. Becomes familiar with properties of non-negative numbers by

   a. Stating in words and symbols the following properties:

      (1) Closure
      (2) Commutative
      (3) Associative
      (4) Identity element
      (5) Inverse element
      (6) Distributive

   b. Identifying properties of non-negative rational numbers from given statements

9. Becomes proficient in operating on non-negative rational numbers by

   a. Adding with three or more fractions having like or unlike denominators

   b. Subtracting with fractions having like or unlike denominators

c. Multiplying with fractions

d. Dividing with fractions

e. Adding with three or more mixed numerals having like or unlike denominators

f. Subtracting with mixed numerals having like or unlike denominators

g. Multiplying with mixed numerals

h. Dividing with mixed numerals

i. Adding with three or more decimals

j. Subtracting with decimals

k. Multiplying with decimals

l. Dividing with decimals

m. Operating on numbers written in different forms such as

$$\begin{array}{r} 342 \\ -\ 66\ \frac{4}{7} \\ \hline \end{array} \quad ; \quad 2\ \frac{1}{3} \times \frac{2}{3}; \quad .00054 \div 60; \quad \frac{5 - \frac{1}{4}}{3 + \frac{1}{2}}; \quad 17\ \frac{1}{2} - 2.4$$

C. Exponents and Expanded Notation

10. Uses exponents to write concise expressions for non-negative rational numbers by

a. Writing $n^0 = 1$ for $n \neq 0$

b. Writing n^2 for $n \times n$ and n^3 for $n \times n \times n$ and vice versa (n is a whole number)

c. Illustrating that if $n \times n = n^2$, then $\sqrt{n^2} = n$ (n is a whole number)

e.g., if $6 \times 6 = 36$, then $\sqrt{36} = 6$

d. Approximating the square root of a given whole number within two whole numbers

e.g., $6 < \sqrt{46} < 7$

e. Evaluating 10^n where n is a non-negative integer

f. Writing in expanded notation any decimal number using non-negative integral exponents

e.g., $75.98 = (7 \times 10^2) + (5 \times 10^1) + (9 \times \frac{1}{10^1}) + (8 \times \frac{1}{10^2})$

D. Ratio, Proportion, and Percent

11. Becomes familiar with the meaning of ratio and proportion by

    a. Using a ratio to compare the number of members in one set with the number of members in another set

    b. Stating that a proportion is a statement of equality of two ratios

12. Acquires a technique for solving a proportion by

    a. Determining if two given ratios are equivalent

    b. Identifying the means and extremes of a given proportion

    c. Illustrating with a given proportion that the product of the means equals the product of the extremes

    d. Solving a given proportion

    e. Solving selected word problems

13. Applies the principles of proportion to percent by

    a. Writing a fraction as a percent and vice versa

    b. Writing a decimal as a percent and vice versa

    c. Computing rate given the percentage and the base

        e.g., 8 is what percent of 64?

    d. Computing the percentage given the rate and the base

        e.g., $\frac{1}{2}$ percent of 182

    e. Computing the base given the percentage and the rate

        e.g., 80 is 120 percent of what number?

    f. Finding a percent of a percent of a number

        e.g., Find 10% of 25% of 200

    g. Computing discount, percentage of discount or original price given any two of the three

    h. Computing simple interest, principal, rate or time given any three of the four

    i. Computing compound interest when given principal, rate and time

j. Computing commission, rate of commission, and sales given any two of the three

E. Geometry

14. Recognizes geometric figures as sets of points in space by

a. Identifying each of the following geometric figures:

(1) point	(8) triangle
(2) line	(9) half-line
(3) plane	(10) half-plane
(4) space	(11) half-space
(5) ray	(12) closed curve
(6) angle	(13) simple closed curve
(7) line segment	(14) quadrilateral

b. Classifying given geometric figures as one-, two-, or three-dimensional

15. Classifies geometric figures according to size and shape by

a. Determining if a given angle is acute, obtuse, or right

b. Determining if a given triangle is scalene, isosceles, or equilateral (equiangular)

c. Determining if a given quadrilateral is a trapezoid, an isosceles trapezoid, a parallelogram, a rectangle, a rhombus, or a square

d. Identifying the congruent ($\cong$) triangle from a given set of triangles

e. Identifying the similar ($\sim$) triangles from a given set of triangles

16. Describes some of the relationships involving two or more lines in a plane by

a. Using the symbols $\parallel$ and $\perp$ to denote parallel lines and perpendicular lines, respectively

b. Naming corresponding angles, alternate interior angles, and interior angles on the same side of the transversal when given two lines and a transversal

c. Stating the relationship between parallel lines and the measures of the following types of angles:

(1) Corresponding angles
(2) Alternate interior angles
(3) Alternate exterior angles
(4) Interior angles on the same side of the transversal

F. Probability

17. Makes reasonable conjectures based on purely chance phenomena by

a. Determining the sample space for a given experiment

b. Determining the events for a given experiment

c. Determining the simple events for a given experiment

d. Listing the possible outcomes for a given experiment where each outcome is an ordered pair

e. Expressing the probability of an event as a fraction between 0 and 1

f. Stating that if an event is impossible, the probability of that event is 0

g. Stating that if an event is certain, the probability of that event is 1

h. Illustrating that the sum of all the simple events of an event space is 1

i. Finding the probability of an event by summing the probabilities of its simple events

G. Measurement

18. Acquires methods for determining error in measurement by

a. Stating that a smaller unit of measure results in a more precise measurement

b. Finding the greatest possible error for a given measurement

c. Finding the percent of error for a given measurement

19. Compares different units of measure in a given system by

a. Converting units of measure within the U.S. system

b. Converting units of measure within the metric system

c. Converting square units of measure within the U.S. system

d. Converting square units of measure within the metric system

e. Adding and subtracting like units of measure

e.g., 12 ft. 4 in. + 5 ft. 9 in. = 18 ft. 1 in.

20. Determines measures of geometric figures by

    a. Using a protractor to find the measure of a given angle

    b. Finding the perimeter of a given polygon

    c. Finding the circumference of a given circle by using formulas

    d. Finding the areas of a given triangle, parallelogram, trapezoid, and circle by using formulas

    e. Estimating the area of a given polygonal region

    f. Finding the area of a polygonal region that is a subset of another polygonal region

    g. Finding the surface area of a given rectangular prism, triangular prism, and right circular cylinder by using formulas

    h. Finding the volume of a given rectangular prism, triangular prism, and right circular cylinder by using formulas

    i. Solving selected word problems

H. Statistics

21. Obtains and reports information contained in data by

    a. Making a frequency distribution chart from given data

    b. Using a frequency distribution chart to find mean, median, mode, and range

    c. Representing data by a histogram

    d. Interpreting and extrapolating information from

        (1) bar graphs
        (2) line graphs
        (3) circle graphs
        (4) pictographs
        (5) histograms

I. Integers

22. Orders integers by

    a. Locating a given integer on the number line

    b. Using the symbols $<$, $=$, or $>$ to order two given integers

23. Becomes proficient in operating on integers by

    a. Adding three or more given integers

    b. Subtracting given integers

    c. Multiplying given integers

    d. Dividing given integers

    e. Finding the average of three or more given integers

    f. Solving selected word problems involving integers

J. Variables

24. Uses variables to represent numbers by

    a. Evaluating a given variable expression using a specified replacement value

        e.g., $3x - 1$; $x = -2$

    b. Evaluating a given formula using specified replacement values

        e.g., $A = \frac{1}{2}h(b_1 + b_2)$; $h = 7$, $b_1 = 13$, $b_2 = 21$

    c. Identifying the like terms from a given list of terms

    d. Combining like terms

    e. Finding the solution set of a given first degree equation in one variable

    f. Checking the solution to a given equation

    g. Solving selected word problems

K. Graphs

25. Relates ordered pairs of integers to points in a coordinate plane by

    a. Identifying the following parts of a rectangular coordinate system:

        (1) x-axis
        (2) y-axis
        (3) origin
        (4) quadrants

b. Naming the first coordinate and the second coordinate of a given ordered pair

c. Writing the ordered pair corresponding to a given point in the coordinate plane

d. Graphing the point corresponding to a given ordered pair of rational numbers

e. Graphing the solution set of a given first degree equation in two variables

General Mathematics I, Grade 8 (1 Credit)

Prerequisite: General Mathematics Grade 7

General Mathematics Grade 8 introduces the student to topics which serve to unify and clarify previously learned topics. The topics, as in Grade 7, should relate to real life experiences.

Unit IV objectives are not minimal but are given as suggestions. The topics in this unit may be replaced by other appropriate topics such as sequences and series or probability depending upon the nature and needs of the students.

Upon completion of General Mathematics Grade 8 the student should have a fundamental knowledge of mathematics needed in society.

GENERAL MATHEMATICS GRADE 8 (180 days)

Units

A. Integers

B. Rational Numbers

C. Geometry

D. Exponents, Expanded Notation and Scientific Notation

E. Real Numbers

F. Measurement

G. Equations, Inequalities and Graphs

H. Consumer Mathematics

I. Modular Arithmetic

J. Numeration Systems

GENERAL MATHEMATICS GRADE 8

A. Integers

1. Becomes familiar with properties of integers by

   a. Stating in words and symbols the following properties of addition and multiplication:

      (1) Closure
      (2) Commutative
      (3) Associative
      (4) Distributive
      (5) Inverse element

   b. Identifying properties of integers from given statements

2. Becomes proficient in operating on integers by

   a. Adding given integers

   b. Subtracting given integers

   c. Multiplying given integers

   d. Dividing given integers

   e. Finding the average of two or more integers

   f. Solving selected word problems involving integers

B. Rational Numbers

3. Orders the rational numbers by

   a. Locating a given rational number on the number line

   b. Writing $<$, $=$, or $>$ between two given rational numbers

4. Becomes familiar with the properties of rational numbers by

   a. Stating in words and symbols the following properties of addition and multiplication:

      (1) Closure
      (2) Commutative
      (3) Associative
      (4) Identity element
      (5) Inverse element
      (6) Distributive

   b. Identifying properties of rational numbers from given statements

5. Becomes proficient in operating on rational numbers by

   a. Adding, subtracting, multiplying, and dividing rational numbers written as fractions or mixed numerals

   b. Adding, subtracting, multiplying, and dividing rational numbers written as decimals

   c. Adding, subtracting, multiplying, and dividing rational numbers written in different forms

      e.g., (1) $5\frac{3}{4} \times 1.75$

      (2) $5.75 \div 2\%$

   d. Using a given order of operations to rename a given numerical expression by a rational number

      e.g., (1) $-6 \div 2 \times 3$

      (2) $\dfrac{3 + \frac{2}{5}}{4 - \frac{3}{5}}$

   e. Solving selected word problems involving rational numbers

C. Geometry

6. Obtains a pictorial representation of geometric figures by constructing with a compass and a straightedge each of the following:

   a. The perpendicular line to a given line from a given point not on the given line

   b. The perpendicular line to a given line at a given point of the given line

   c. A line parallel to a given line through a given point not on the given line

   d. Geometric figures congruent to each of the following:

      (1) a given line segment
      (2) a given angle
      (3) a given circle

   e. A regular polygon with a given number of sides that is inscribed in or circumscribed about a given circle

f. An equilateral triangle

g. A square

h. A regular hexagon

i. A regular pentagon

j. A triangle congruent to a given triangle using each of the following conditions:

(1) Side Side Side
(2) Side Angle Side
(3) Angle Side Angle

k. A median and an altitude for a given triangle

7. Classifies angles according to size and position by

a. Stating the definition of complementary angles

b. Finding the measure of the complement of a given angle

c. Stating the definition of supplementary angles

d. Finding the measure of the supplement of a given angle

e. Identifying two given angles as being complementary, supplementary, or neither

f. Identifying the vertical angles from a given set of angles

g. Identifying the adjacent angles from a given set of angles

h. Finding, when sufficient information is given, the measures of angles formed by two parallel lines and a transversal such as

(1) corresponding angles
(2) alternate interior angles
(3) alternate exterior angles
(4) interior angles on the same side of the transversal

D. Exponents, Expanded Notation, and Scientific Notation

8. Uses exponents when representing rational numbers by

a. Raising a given integer to a given non-negative integral power

e.g., $(-2)^3 = -8$

b. Writing a given integer as a power

e.g., $81 = 3^4$

c. Multiplying and dividing given powers with the same base

e.g., $7^5 \cdot 7^3 = 7^8$ and $7^5 \div 7^3 = 7^2$

d. Evaluating a given numerical expression involving powers

e.g., $3^2 \cdot 2^3$

9. Acquires a technique for expressing very large or very small numbers by

a. Expressing 10^{-m} as $\frac{1}{10^m}$ and vice versa where m is a positive integer

b. Writing a given decimal in expanded notation using integral exponents

e.g., $36.75 = (3 \times 10^1) + (6 \times 10^0) + (7 \times 10^{-1}) + (5 \times 10^{-2})$

c. Writing a given decimal in scientific notation using integral exponents

e.g., $.0000082 = 8.2 \times 10^{-6}$

d. Multiplying and dividing given numbers written in scientific notation

e. Solving selected word problems involving scientific notation

E. Real Numbers

10. Becomes aware of the set of irrational numbers by

a. Finding the solution to a sentence such as $\sqrt{a} \doteq \square$ (where a is a positive integer) by using an approximation method and by a table

b. Finding the square of a given number by solving a sentence such as $(3.12)^2 = \square$ by using an approximation method and by a table

c. Associating irrational numbers with nonterminating, nonrepeating decimals

d. Recognizing that the square root of a nonperfect square is an irrational number

e. Giving examples of other irrational numbers

e.g., π, $\sqrt[3]{10}$, $\sqrt[4]{8}$

f. Locates a given irrational number, such as $\sqrt{2}$, on the number line

F. Measurement

11. Finds the error in a given measurement by

a. Determining which digits in a given measurement are significant

b. Finding the precision of a given measurement

c Finding the greatest possible error of a given measurement

d. Finding the relative error of a given measurement

e. Determining the accuracy of a given measurement

12. Determines measures of certain geometric figures by

a. Finding the circumference and the area of a circle with the formulas

b. Finding the surface area of simple closed surfaces (regular prisms, cones, pyramids, right circular cylinders, and spheres) with the formulas

c. Finding the volume of a solid region (pyramids, spheres, and cones) with the formulas

d. Finding the length of a missing side of a right triangle with the Pythagorean theorem

e. Finding the length of corresponding sides of similar triangles using proportions

f. Using the formula to find distance, rate, or time ($d = rt$)

g. Solving word problems involving measurement

G. Equations, Inequalities, and Graphs

13. Obtains a readiness for solving equations by

a. Illustrating that a mathematical expression may be numerical or variable

b. Identifying like terms

c. Combining like terms using the distributive property

d. Stating in words and symbols the addition property of equality for rational numbers: If $a = b$, then $a + c = b + c$

e. Stating in words and symbols the multiplication property of equality for rational numbers: If $a = b$, then $ac = bc$

14. Solves equations by

a. Finding solution sets for equations in the form $x + d = b$ using the addition property of equality (x is a variable; d and b are rational numbers)

b. Finding solution sets for equations in the form $ax = b$ using the multiplication property of equality (x is a variable; a and b are rational numbers)

c. Finding the solution sets for equations in the form $ax + b = c$ using the addition and multiplication properties of equality (x is a variable; a, b, and c are rational numbers)

15. Obtains a readiness for solving inequalities by

a. Stating in words and symbols the addition property of inequality for rational numbers: If $a < b$, then $a + c < b + c$

b. Stating in words and symbols the multiplication property of inequality for rational numbers:

If $a < b$ and $c > 0$, then $ac < bc$

If $a < b$ and $c < 0$, then $ac > bc$

If $a < b$ and $c = 0$, then $ac = bc$

16. Solves inequalities by

a. Finding solution sets for inequalities in the form $x + d < b$ using the addition property of inequality

b. Finding solution sets for inequalities in the form $ax < b$ using the multiplication property of inequality

c. Finding the solution sets for equations in the form $ax + b < c$ using the addition and multiplication properties of inequality

17. Depicts sets of ordered pairs of rational numbers by

a. Plotting the points for a given set of ordered pairs

b. Writing the coordinates for a given point

c. Graphing a given rational number on the number line

d. Graphing the solution set of a first degree equation in one variable on the number line

e. Graphing the solution set of a first degree inequality in one variable on the number line

f. Graphing first degree equations in two variables in the coordinate plane

g. Graphing first degree inequalities in two variables in the coordinate plane

h. Graphing compound sentences in one variable with "or" and "and" on the number line

H. Consumer Mathematics

18. Uses mathematics to determine costs by

a. Estimating the cost of several items using mental arithmetic

b. Estimating sales tax on a given purchase

c. Estimating the amount of change from a given purchase

d. Comparing the cost of two or more items using unit pricing

e. Computing the interest on a given item purchased at two given rates and comparing the results

19. Keeps a sample checking account by

a. Filling out a sample checking deposit form

b. Writing a sample check

c. Reconciling a sample bank statement

I. Modular Arithmetic

20. Performs the operations of arithmetic on the elements of a finite system by

a. Identifying the numerals to be used in a given modular system

b. Writing a given whole number as a number in a given modulo (2 through 12)

c. Constructing an addition and multiplication table for a given modulo (2 through 12)

d. Adding, subtracting, multiplying and dividing whole numbers in a given modulo (2 through 12)

e. Determining if the following properties for addition and multiplication hold in a given modulo (2 through 12):

(1) Closure
(2) Commutative
(3) Associative
(4) Identity element
(5) Inverse element
(6) Distributive

21. Illustrates modular systems in life

e.g., (1) hours of the day
(2) days of the week
(3) months of the year

J. Numeration Systems

22. Performs operations of arithmetic in bases other than 10 by

a. Reading and writing numerals in other bases (2 through 12)

b. Counting (1 through 50) in a given base other than 10

c. Writing a numeral given in a base other than 10 as a base 10 numeral

d. Writing a base 10 numeral as a numeral in a given base

e. Adding, subtracting, and multiplying numbers in a given base other than 10

Learning Objectives

The learning objectives are provided as a resource for teachers to use in planning and organizing their program.

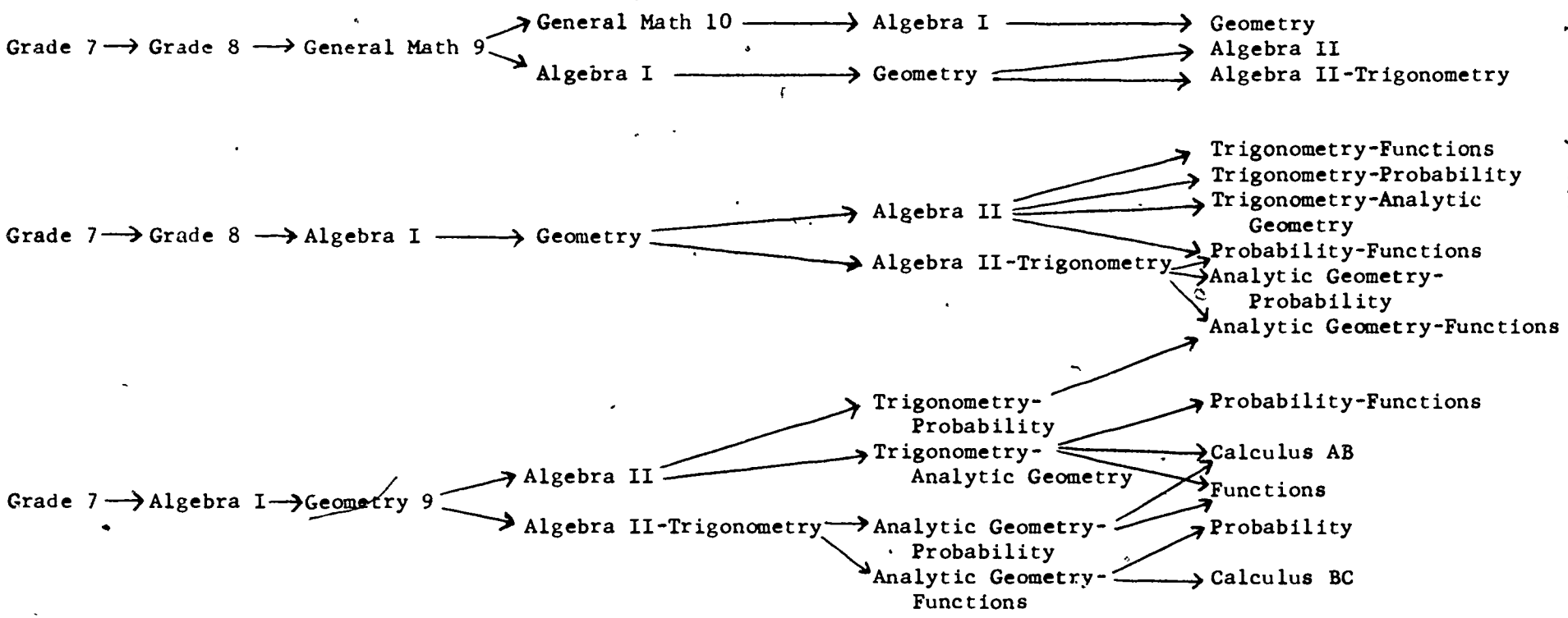
The learning objectives are not all inclusive and may be supplemented or adapted by teachers as a basis for establishing the objectives of their program. If an objective is stated for one level, it is usually not repeated in a succeeding level. Thus, objectives in preceding and succeeding levels are vital to the objectives in each level.

Secondary Objectives

The secondary objectives are organized by courses. For each course, the course name, code, prerequisites, and description are stated. The objectives are written in sequential units. No time limit is suggested for any one unit.

Preceding the course objectives is a sequence chart. The chart may be used in planning an appropriate secondary mathematics program.

SEQUENCES OF COURSES



Computer Science 1 - a one semester course that can be taken after the successful completion of Algebra I.

Computer Science 2 - a one year course that can be taken after or in conjunction with Algebra II or Algebra II-Trigonometry.

Senior Mathematics - a course designed to offer opportunities to concentrate on selected topics.

Consumer Mathematics - a course designed to help the student become a judicious consumer that can be taken by 10th grade students (after successful completion of Geometry) or 11th and 12th grade students.

Advanced Placement Sequences

In mathematics two advanced placement sequences are offered for students who desire to receive college credit for mathematics content that is completed in high school. Those students who take Calculus BC can earn up to one full year of credit, while students who take Calculus AB can earn one semester credit.

Comprehensive examinations for Calculus AB and Calculus BC are administered by the College Entrance Examination Board to determine the amount of college credit that is earned.

Grade 7
↓
Algebra I
↓
Geometry
↓
Algebra II-Trigonometry
↓
Analytic Geometry-Functions
↓
Calculus BC

Grade 7
↓
Algebra I
↓
Geometry
↓
Algebra II
↓
Trigonometry-Analytic Geometry
↓
Calculus AB

Career Mathematics

The academic career related courses beyond geometry are:

Trigonometry
Computer Science I
Computer Science II
Probability and Statistics
Consumer Mathematics

General Mathematics II, Grade 9 (1 Credit -- General Mathematics)

There are no minimum requirements for General Mathematics II, Grade 9. The students enrolled in this course usually have not completed level 7 objectives. Diagnostic techniques should be used to determine each student's level, and every effort should be made to enable the student to raise his or her level. Emphasis should be placed upon preparing the student for dealing with real life situations. A variety of topics and approaches is essential to achieving this goal. The objectives given are all optional and are designed as examples for applying mathematics to real life situations and for offering the necessary variety of topics.

Upon completion of General Mathematics II, Grade 9 the student should

1. Complete level 7 objectives
2. Achieve success in basic mathematics
3. Relate mathematics to real life situations

Mathematics 9 (3121)
Section B
September 3, 1974

GENERAL MATHEMATICS II, GRADE 9 (180 days)

Units

A. Owning and Operating a Car

B. Cross Country Trip

C. Budgets

D. Banking and Interest

E. Taxation

F. Investments

G. Probability

H. Insurance

I. Graphs

J. Measurement

K. Geometry

L. Modular Arithmetic (Clock Arithmetic)

M. Introduction to Algebra

GENERAL MATHEMATICS II, GRADE 9

A. Owning and Operating a Car

1. Increases proficiency in operating on whole numbers, decimals and fractions by

   a. Computing driving costs using cost per gallon of gasoline and rates per mile

   b. Finding the cost of buying a new car including licenses, insurance and sales tax

   c. Determining costs of owning a car including insurance, licenses, gasoline, oil, depreciation, repairs and tires

   d. Finding stopping distances for various speeds

   e. Solving selected word problems

2. Discusses the responsibilities of an automobile owner by

   a. Identifying ways to improve automobile safety; i.e., seat belts, tires, equipment replacement

   b. Illustrating ways to increase automobile performance and appearance; i.e., tune-ups, waxing, seat covers

B. Cross Country Trip

3. Develops his ability to use scale drawings by

   a. Reading maps

   b. Approximating distances from maps

   c. Describing several possible routes between two cities

   d. Determining the shortest route between two cities

4. Increases proficiency in operating on decimals by

   a. Determining expenses involved in a trip

      e.g., tolls, gasoline, motels, meals, campground fees

   b. Planning a long distance trip including description of the route and necessary expenditures

C. Budgets

5. Recognizes the need for accurate record keeping by

a. Estimating expenses over a certain period of time

b. Keeping an account of actual expenses over that time

c. Comparing estimated expenses with actual expenses

d. Classifying given family expenses as household, food, clothing, savings, life insurance, transportation, health, charities, recreation

e. Solving selected word problems

D. Banking and Interest

6. Uses operations on percents and decimals in situations involving money by

a. Finding the interest on a given principal at a given rate over a given period of time

b. Using the formula I = prt to determine the unknown when given three of the variables

c. Operating a sample checking account

(1) Writing a sample check
(2) Filling out a deposit slip
(3) Balancing a bankbook
(4) Reconciling a bank statement

d. Operating a sample savings account

(1) Making a deposit
(2) Filling out a withdrawal form
(3) Finding the interest incurred

E. Taxation

7. Becomes aware of taxes at the local, state and federal levels by

a. Describing the reasons for taxes

b. Identifying the following taxes as being local, state or federal:

(1) Sales
(2) Income
(3) Social Security
(4) Property
(5) Gasoline
(6) Corporation
(7) Customs
(8) Alcohol
(9) Automobile
(10) School

c. Approximating personal income taxes (federal, state, social security) using the following terms:

(1) Income
(2) Taxable income
(3) Exemptions for dependents
(4) Standard deduction

d. Solving problems involving gasoline taxes

e. Finding sales tax on items purchased

f. Solving problems involving personal property taxes

F. Investments

8. Uses operations on percents and decimals in situations involving investment by

a. Becoming familiar with terms needed to invest in U.S. Savings Bonds; i.e., maturity value, issue price

b. Finding cost (issue price) and interest at maturity of various savings bonds

c. Solving problems involving interest on loans

d. Finding interest on bank savings account given the principal, time and rate

e. Using basic terms in describing the stock market

f. Observing and recording progress of one stock over a given period of time

g. Explaining some differences between a bank and a credit union

G. Probability

9. Uses fractions in experimentation and prediction of possible outcomes of chance events by

a. Predicting outcomes of a given number of coin tosses and dice rolls

b. Recording the results of a given number of coin tosses and dice rolls

c. Testing hypotheses by further experimentation

d. Writing the probability of a simple event as a fraction between 0 and I $\left(\frac{\text{\# successful ways}}{\text{\# possible ways}}\right)$

e. Finding the probability of simple events

f. Finding probabilities involving cards

g. Applying probability ratios to real life decision making

h. Using a sample space with sample points as an alternative method in determining probabilities involving two elements

i. Multiplying the probability of each event to find the probability of two dependent events

j. Finding the probability of an event not occurring by subtracting from 1 the probability of its occurring

H. Insurance

10. Develops familiarity with the various types of insurance by

a. Stating how probability affects insurance companies and their policies

b. Identifying different kinds of insurance

e.g., auto, life, fire, health, social security

c. Finding rates and premiums of insurance policies given sufficient information

d. Identifying life insurance company requirements; i.e., medical exams, nature of employment, age

e. Solving selected word problems involving insurance

I. Graphs

11. Obtains information which is represented pictorially by identifying and interpreting the following types of graphs:

(1) Pictograms
(2) Bar graphs
(3) Line graphs
(4) Circle graphs

12. Displays information contained in a given set of data by

a. Constructing horizontal and vertical bar graphs

b. Drawing a pictogram using symbols to represent numbers

c. Constructing a circle graph when given the relationship of each part to the whole

d. Plotting and connecting points to form pictures when given the rectangular coordinates

e. Constructing line graphs

J. Measurement

13. Becomes aware of the standard units of measure in different systems by

a. Performing the operations of arithmetic on the following measures:

(1) Length
(2) Liquid measure
(3) Dry measure
(4) Weight
(5) Time
(6) Area
(7) Volume
(8) Angles and arcs
(9) Temperature

b. Converting units of measure in the U.S. system

c. Converting units of measure in the metric system

d. Estimating measures of physical objects found in the classroom

e. Solving selected word problems involving measurement

K. Geometry

14. Identifies and classifies geometric figures according to shape and size by

a. Labeling points, lines, line segments, rays and angles

b. Classifying a given angle as acute, right or obtuse

c. Stating that the sum of the measures of the angles on one side of a line is 180

d. Describing intersecting, parallel and perpendicular lines

e. Identifying the following geometric figures:

(1) Cone
(2) Cube
(3) Rectangular solid
(4) Cylinder
(5) Sphere
(6) Pyramid

f. Making triangles, rectangles, parallelograms, trapezoids and hexagons by using objects such as tangrams

g. Classifying a given polygon as a (n)

(1) Triangle
(2) Quadrilateral
(3) Pentagon
(4) Hexagon
(5) Septagon
(6) Octagon

h. Constructing polygons using various materials such as pencil and paper, string, straws, cardboard

i. Finding perimeters and areas of rectangles, parallelograms, triangles, and squares

L. Modular Arithmetic (Clock Arithmetic)

15. Applies arithmetic skills to a finite mathematical system by

a. Using notation for modular arithmetic

e.g., $8 \pmod 5 = 3$

b. Adding and multiplying two numbers in a given modulo

c. Illustrating how modular arithmetic applies to real life situations; i.e., days of the week, hours on the clock, months in a year

d. Solving selected word problems involving a modular system

M. Introduction to Algebra

16. Generalizes arithmetic processes through the use of variables by

a. Writing algebraic expressions containing one or more variables

b. Evaluating a given algebraic expression using a given replacement

e.g., $3x \div 5$, $x = 1$

c. Solving a given mathematical sentence in one variable

d. Solving equations with a given replacement set

e. Graphing the solution set of a given mathematical sentence on a number line

f. Translating verbal statements into mathematical sentences

Algebra I, Part I (1 Credit -- General Mathematics)

Prerequisite: General Mathematics I, Grade 8

Algebra I, Part I is designed to enable a student to take a full-year course in the first half of Algebra I; hence, the course progresses at a slower pace than Algebra I. The objectives of Units I and II of Algebra I should be completed.

ALGEBRA I, PART I (180 days)

Units

A. Exponents

B. Open Expressions

C. Equations in One Variable

D. Inequalities

E. Graphs in a Cartesian Plane

F. Systems of Two Linear Sentences in Two Variables

Algebra I, Part II (1 Credit -- Academic Mathematics)

Prerequisite: Algebra I, Part I

Algebra I, Part II is designed to enable a student to take a full-year course in the second half of Algebra I. The objectives of Units III and IV of Algebra I should be completed.

ALGEBRA I, PART II (180 days)

Units

G. Polynomials and Operations

H. Factoring Polynomials

I. Polynomial Equations

J. Rational Expressions

K. Radicals and Irrational Numbers

L. Quadratic Equations

Algebra I (1 Credit -- Academic Mathematics)

Prerequisite: General Mathematics I, Grade 8

Algebra I is the first course in the college preparatory program. Emphasis is placed on informal justification of processes through the properties of real numbers. Word problems are used as a tool for learning the applications of algebraic techniques.

Upon completion of Algebra I the student should begin to

1. Understand that algebraic expressions are real numbers
2. Confirm his intuitions and build confidence for future mathematical studies
3. Increase his ability to reason at an abstract level

Algebra I (3130)
Section B
September 3, 1974

ALGEBRA I (180 days)

Units

A. Exponents

B. Open Expressions

C. Equations in One Variable

D. Inequalities

E. Graphs in a Cartesian Plane

F. Systems of Two Linear Sentences in Two Variables

G. Polynomials and Operations

H. Factoring Polynomials

I. Polynomial Equations

J. Rational Expressions

K. Radicals and Irrational Numbers

L. Quadratic Equations

ALGEBRA I

A. Exponents

1. Writes concise, readable expressions using exponential notation by

   a. Identifying the coefficient, base and exponent of a given power

   b. Multiplying given powers with the same base using the property $x^m \cdot x^n = x^{m+n}$ (m and n are positive integers)

   c. Raising a given power to a power using the property $(x^m)^n = x^{mn}$ (m and n are positive integers)

   d. Raising a given product to a power using the property $(xy)^m = x^m y^m$ (m is a positive integer)

   e. Raising a given quotient to a power using the property $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ ($y \neq 0$; m is a positive integer)

   f. Dividing two given powers with the same base using the property $\frac{x^m}{x^n} = x^{m-n}$ ($x \neq 0$; m and n are positive integers)

   g. Stating the definition of x^0 ($x \neq 0$); i.e., $x^0 = 1$ ($x \neq 0$)

   h. Stating the definition of x^{-m} (m is a positive integer); i.e., $x^{-m} = \frac{1}{x^m}$ (m is a positive integer)

   i. Simplifying a given expression involving powers using more than one of the properties in objectives 1b through 1h

      e.g., (1) $(3x^2y)^3 (2xy^2)$

      (2) $\frac{4x^3}{(2x)^4}$

      (3) $(4x^{-3}y^2)(x^2y^{-5})$

B. Open Expressions

2. Recognizes that an open expression represents a real number by evaluating a given open expression using specified replacement values.

   e.g., $4(5x - 2y)$ when $x = 2$ and $y = -1$

3. Writes open expressions in a usable form for given situations by

   a. Identifying the terms of a given open expression

   b. Selecting like (similar) terms from a given list of terms

   c. Adding and subtracting like terms

   d. Multiplying a given open expression by a constant using the distributive property

      e.g., $5(2x - 1)$

   e. Finding the opposite of a given open expression

      e.g., $-(3y - 5)$

   f. Simplifying a given open expression by using the distributive property to eliminate parentheses and by combining like terms

      e.g., $3(2x - y) - (4x + 2y)$

C. Equations in One Variable

4. Determines solutions of equations in one variable using sets of equivalent equations by

   a. Solving a given equation in the form $ax + b = c$ using the following properties of equality:

      (1) $x = y$, then $x + a = y + a$

      (2) $x = y$, then $x - a = y - a$

      (3) $x = y$, then $ax = ay$

      (4) $x = y$, then $\frac{x}{a} = \frac{y}{a}$ when $a \neq 0$

   b. Solving a given equation in the form $ax + b = cx + d$ using the properties of equality

   c. Solving a given equation by simplifying one or both members and using the properties of equality

      e.g., $2(3x + 1) - 4x = 8$
      $5x - 3 + 2x = 3 - 4x + 8$

   d. Stating the definition of the absolute value of any real number

      e.g., $|x| = x, \; x \geq 0$

      $|x| = -x, \; x < 0$.

e. Writing a compound sentence that is equivalent to a given equation involving absolute value

e.g., problem $|x - 3| = 5$; answer $x - 3 = 5$ or $x - 3 = -5$

f. Solving a given equation involving absolute value which can be written in the form $|ax + b| = c$

5. Applies mathematics to selected real life situations by

a. Translating given verbal phrases to mathematical expressions

b. Translating given verbal sentences to mathematical equations

c. Solving selected word problems

D. Inequalities

6. Obtains the necessary notation for describing the order of real numbers by reading, writing and interpreting the following symbols of inequality:

$<, >, \leq, \geq, \not>, \not<, \neq$

7. Determines solutions of inequalities using sets of equivalent inequalities by

a. Solving a given inequality in the form $ax + b < c$ using the following properties of inequality:

(1) $x < y$, then $x + a < y + a$

(2) $x < y$, then $x - a < y - a$

(3) $x < y$, then $ax < ay$ when $a > 0$

$ax = ay$ when $a = 0$

$ax > ay$ when $a < 0$

(4) $x < y$, then $\frac{x}{a} < \frac{y}{a}$ when $a > 0$

$\frac{x}{a} > \frac{y}{a}$ when $a < 0$

b. Solving a given inequality of the form $ax + b < cx + d$ using the properties of inequality

(Objective 7a and 7b should be extended to include the $>$, $\leq$ and $\geq$ relations.)

c. Solving a given inequality by simplifying one or both members and using the properties of inequality

e.g., $3(2x - 1) - 5x \geq 7$

$2x - 3 + 5x > x + 12 + 7x$

d. Writing a compound inequality equivalent to a given inequality involving absolute value

e.g., problem $|2x + 1| < 5$; answer $-5 < 2x + 1 < 5$

problem $|3x - 4| > 2$; answer $3x - 4 > 2$ or $3x - 4 < -2$

e. Solving a given inequality involving absolute value which can be written in the form $|ax + b| < c$

(Objective 7e should be extended to include the $>$, $\leq$ and $\geq$ relations.)

8. Graphs the solution of a given first degree compound inequality in one variable

e.g., (1) $2 \leq x < 6$

(2) $x < -1$ or $x \geq 3$

E. Graphs in a Cartesian Plane

9. Displays the solution set of a first degree sentence in two variables by

a. Graphing the solution set of a given equation which can be written in the form $Ax + By + C = 0$

b. Graphing the solution set of a given inequality which can be written in the form $Ax + By + C < 0$ or $Ax + By + C \leq 0$

10. Describes some of the characteristics of a line by

a. Determining the slope of a selected line given

(1) the coordinates of two points on the line
(2) its graph
(3) its equation

b. Determining if a line has a positive, negative, zero or no slope from a graph

c. Writing a given linear equation in the form $y = mx + b$ and recognizing that m represents the slope of the line and b represents the y-intercept of the line

11. Writes the equation of a line given

(1) the slope and y-intercept of the line
(2) the slope and the coordinates of a point of the line
(3) the coordinates of two points of the line

F. Systems of Two Linear Sentences in Two Variables

12. Displays a system of two first degree sentences in two variables by

a. Graphing a given system of linear equations (including inconsistent, dependent and independent systems)

b. Graphing the union or the intersection of the solution sets of two given linear inequalities

13. Uses an equation in one variable to obtain the solution of a given system of two equations in two variables by

a. Solving a given system of linear equations by the addition or subtraction methods

b. Solving a given system of linear equations by the substitution method

14. Solves selected word problems using a system of equations

G. Polynomials and Operations

15. Classifies expressions involving powers by

a. Identifying a given polynomial as a monomial, a binomial or a trinomial

b. Finding the degree of a given polynomial

16. Recognizes that polynomials represent real numbers and can be operated on by

a. Adding and subtracting two given polynomials

e.g., $(3x^2 + 2x) + (6x + 3)$

$2x^3 - (3x^3 - 6x)$

$(5x^2y + 6xy) + (3xy + 4xy^2)$

b. Multiplying two given polynomials

e.g., $3x(x^3 - 2x + 6)$

$(x - 2)(x + 5)$

$(x + 2y)(3x - y)$

c. Finding the quotient of two given polynomials where the degree of the divisor is less than the degree of the dividend

H. Factoring Polynomials

17. Expresses polynomials in forms conducive to problem solving by

a. Writing the prime factorization of a given monomial

b. Finding the greatest common factor of a given set of monomials

c. Selecting the greatest common factor of the terms in a given polynomial and then writing the polynomial as the product of this common factor and another polynomial

d. Factoring a given polynomial that can be written as the difference of two squares

e. Factoring a given trinomial that is a perfect square

f. Factoring a given trinomial of the form $ax^2 + bx + c$ where $a = 1$, b and c are nonzero integers

g. Factoring a given trinomial of the form $ax^2 + bx + c$ where a, b, and c are nonzero integers

h. Writing the prime factorization of a given polynomial.

e.g., $2x^2 - 2y^2$

$9x^3 - 12x^2 + 3x$

I. Polynomial Equations

18. Determines solutions of polynomial equations using compound first degree equations by

a. Solving a given polynomial equation of the form $(ax + b)(cx + d) = 0$ using the property $a \cdot b = 0$ if and only if $a = 0$ or $b = 0$

b. Solving a given polynomial equation that can be written in the form $ax^2 + bx + c = 0$ where $ax^2 + bx + c$ is a factorable polynomial with a, b, and c as integers

J. Rational Expressions

19. Applies patterns in operations on rational numbers to operations on rational expressions by

a. Determining the restrictions on the variable in a given rational expression

b. Writing a given rational expression in lowest terms

c. Multiplying and dividing two given rational expressions

d. Adding and subtracting two given rational expressions with like denominators

e. Adding and subtracting two given rational expressions with unlike denominators

f. Solving a given equation with fractional coefficients

g. Solving a given equation involving rational expressions

K. Radicals and Irrational Numbers

20. Uses notation for writing irrational numbers in a concise form by

a. Identifying the radicand, radical sign, and index of a given radical

b. Writing the exponential form of a given radical

e.g., problem $\sqrt[3]{x^2}$; answer $x^{2/3}$

c. Writing the radical form of a given expression involving rational number exponents

e.g., problem $x^{5/4}$; answer $\sqrt[4]{x^5}$

d. Writing the squares of the integers from 1 through 25

e. Writing each principal square root and negative square root of the positive integers through 625 which are perfect squares

f. Approximating the principal square root of a positive integer less than 100

g. Stating that the square root of a positive integer that is not a perfect square is a nonterminating, nonrepeating decimal

i.e., an irrational number

h. Simplifying a given radical by using the properties ($a \geq 0$, $b > 0$):

(1) $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

(2) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

21. Recognizes that radicals represent real numbers and can be operated on by

a. Multiplying and dividing two given radicals

b. Writing a given radical expression with a rational denominator

c. Adding and subtracting two given radicals

d. Solving a given radical equation

L. Quadratic Equations

22. Determines solutions of a quadratic equation

23. Acquires alternative techniques for solving quadratic equations by

a. Completing the square to solve a given quadratic equation

b. Using the quadratic formula to solve a given quadratic equation

Geometry (1 Credit -- Academic Mathematics).

Prerequisite: Algebra I or Algebra I, Part II

Geometry is the study of a mathematical structure whose basic elements are sets of points. Plane, solid, and coordinate geometry are usually treated both geometrically and algebraically. Proofs are emphasized throughout Units I and II, and applications of theorems throughout Units III and IV.

Upon completion of Geometry the student should be able to

1. Draw logical conclusions from a given hypothesis
2. Write mathematical proofs
3. Apply algebraic techniques to geometric problems
4. Use measurement to describe geometric figures

GEOMETRY (180 days)

Units

A. Points, Lines and Planes

B. Angles and Triangles

C. Perpendicular Lines in the Plane

D. Inequalities in Triangles

E. Perpendicular Lines and Planes in Space

F. Parallel Lines, Quadrilaterals and Right Triangles

G. Areas and Polygonal Regions

H. Proportionality and Similarity

I. Coordinate Geometry

J. Circles

K. Areas of Circles and Regular Polygons

L. Solids

M. Volumes of Solids

GEOMETRY

A. Points, Lines and Planes

1. Describes points on a line by

   a. Stating the following order properties for real numbers:

      (1) Trichotomy
      (2) Transitive

   b. Determining the distance between two given points on a number line by taking the absolute value of the difference of the corresponding coordinates

   c. Determining which of three given points is between the other two

   d. Describing and writing symbols for

      (1) Line segment
      (2) Ray
      (3) Opposite rays
      (4) Line

   e. Writing the definition of the midpoint of a line segment

2. Describes points, lines and planes in space by

   a. Stating necessary and sufficient conditions to determine a unique line and a unique plane

   b. Describing a given set of points as

      (1) Collinear
      (2) Noncollinear, but complaner
      (3) Noncomplaner

   c. Determining if a given set of points is a convex set

   d. Writing the intersection and union of two or more given sets of points

   e. Describing the convexed sets that are formed when

      (1) A point separates a line
      (2) A line separates a plane
      (3) A plane separates space

B. Angles and Triangles

3. Describes points in a plane by

a. Stating the definition of an angle and the definition of a triangle

b. Determining if a given point lies in the interior or exterior of a given angle

c. Determining if a given point lies in the interior or exterior of a given triangle

4. Classifies and compares angles by

a. Stating the correspondence which exists between an angle and a real number

b. Stating the relationship between two angles being supplementary and forming a linear pair

c. Finding the measure of the complement and supplement of a given angle

d. Stating the relationship between perpendicular lines and right angles

e. Stating the definition of congruent angles

f. Solving problems involving angular measure given that a point D is in the interior of an angle BAC; i. e., $m\angle BAD + m\angle DAC = m\angle BAC$

5. Begins to use deductive reasoning in constructing proofs by

a. Stating how undefined terms, definitions, postulates and theorems are used to prove theorems

b. Identifying the hypothesis (given) and conclusion (proven) of a given statement

c. Proving that two triangles are congruent using

(1) Side, Angle, Side
(2) Angle, Side, Angle
(3) Side, Side, Side

d. Making reasonable conjecture for a given geometric statement

e. Finding for a given statement the converse, the inverse and the contrapositive

f. Classifying as true or false a given statement and its converse

g. Stating the relationship between two angles of a triangle given that two sides are congruent and vice versa

h. Stating the relationship involving the three angles of a triangle given that three sides are congruent and vice versa

i. Determining and drawing the necessary auxiliary lines needed to prove a given theorem

j. Writing selected proofs

6. States the definitions of

   a. An angle bisector

   b. A median of a triangle

   c. An altitude of a triangle

C. Perpendicular Lines in a Plane

7. Illustrates the existence and uniqueness of perpendicular lines in a plane by

   a. Identifying among a given set of proof those which are existence and/or uniqueness proofs

   b. Stating the conditions that determine a unique line perpendicular to a given line through a given point

   c. Solving problems involving the theorem which states that the perpendicular bisector of a segment in a plane is the set of all points in the plane equidistant from the endpoints of a segment

D. Inequalities in Triangles

8. Orders angles and sides of triangles by

   a. Stating the relationship between an exterior angle of a triangle and the two remote interior angles

   b. Distinguishes between types of proof (direct and indirect)

   c. Writing the sides of a triangle in increasing order given the measures of its angles and vice versa

   d. Determining the possible measures of one side of a triangle given the measures of the other two sides

   e. Stating the definition of the distance from a point to a line

9. Proves two given triangles are congruent using

   a. Side, Angle, Angle

   b. Hypotenuse - Leg

E. Perpendicular Lines and Planes in Space

10. Extends the concept of perpendicularity to space by

a. Stating the definition of a line perpendicular to a plane

b. Writing proof using the theorem that states if a line is perpendicular to each of two intersecting lines at their point of intersection, then it is perpendicular to the plane that contains them

c. Stating conditions to determine the existence and uniqueness of

(1) A line perpendicular to a given plane
(2) A plane perpendicular to a given line

F. Parallel Lines, Quadrilaterals and Right Triangles

11. Shows how certain conditions relate to parallelism by

a. Classifying a given set of lines as intersecting, parallel or skew

b. Stating and using to solve problems the Parallel Postulate; i.e., through a given external point there is only one parallel to a given line

c. Proving the relationship between two lines cut by a transversal and each of the following:

(1) Alternate interior angles
(2) Corresponding angles
(3) Interior angles on the same side of the transversal

d. Stating the definition of the distance between two parallel lines

12. Determines the relationship among the angles of a triangle by

a. Proving that the sum of the measures of the angles of a triangle is 180°

b. Finding the measure of an angle of a triangle given the measures of the other two angles

c. Finding the measure of an angle of an equilateral triangle

d. Finding the measures of two angles of an isosceles triangle given the measure of the other angle

13. Describes quadrilaterals and their properties by

a. Determining if a given four-sided figure is a quadrilateral

b. Identifying the properties of the following quadrilaterals:

(1) Trapezoid
(2) Parallelogram
(3) Rhombus
(4) Rectangle
(5) Square

c. Classifying a quadrilateral with given conditions as a trapezoid, parallelogram, rhombus, rectangle or square

14. Uses for problem solving the following:

a. The median to the hypotenuse of a right triangle is half of the length of the hypotenuse

b. If one angle of a right triangle measures 30, the the side opposite it is half as long as the hypotenuse

G. Areas and Polygonal Regions

15. Solves problems involving areas of the following types of polygonal regions:

a. Square

b. Rectangle

c. Right Triangle

d. Triangle

e. Trapezoid

f. Parallelogram

g. Rhombus

16. Determines relationships among sides of right triangles by

a. Stating the Pythagorean Theorem and its converse

b. Finding the length of a side of a right triangle given the lengths of the other two sides using the Pythagorean Theorem

c. Solving problems involving the relationship between the measures of angles and the lengths of sides in

(1) A 30-60-90 triangle
(2) An isosceles right triangle

H. Proportionality and Similarity

17. Determines measures of geometric figures with the same shape by

a. Finding the geometric mean and the arithmetic mean of two given nonnegative rational numbers

b. Stating the definition of similar triangles

c. Solving proportions determined by a line intersecting two sides of a given triangle and parallel to the third side

d. Writing proofs involving similar triangles using the following:

(1) Angle, Angle
(2) Side, Angle, Side
(3) Side, Side, Side

e. Solving proportions determined by the altitude to the hypotenuse of a given right triangle

f. Writing the ratio of the areas of two similar triangles given the ratio of corresponding sides, altitudes or medians and vice versa

g. Finding the area of one of two similar triangles given the area of the other triangle and the ratio of corresponding sides

I. Coordinate Geometry

18. Uses algebraic techniques to make certain geometric concepts more meaningful by

a. Determining the distance between two points with given coordinates using the distance formula

b. Determining the area and the perimeter when given the coordinates of the vertices of the following regions

(1) Triangle with at least one side parallel to a major axis
(2) Parallelogram with at least one pair of sides parallel to a major axis

c. Determining if two given lines are parallel, perpendicular or neither by comparing their slopes

d. Writing the coordinates of the midpoint of a line segment given the coordinates of its endpoints

e. Classifying a triangle as isosceles, equilateral or right angle given the coordinates of its vertices

f. Classifying a quadrilateral as a trapezoid, parallelogram, rhombus, rectangle or square given the coordinates of its vertices

g. Writing the slope of a line and the coordinates of a point on the line given its equation in the point-slope form; i.e., $y - y_1 = m(x - x_1)$

h. Writing the point-slope form of the equation of a line given

(1) The slope and the coordinates of a point of the line
(2) The coordinates of two points of the line
(3) The coordinates of a point of the line and the equation of a parallel or perpendicular line

J. Circles

19. Describes and measures segments, lines, angles and arcs as they relate to circles by

a. Stating the relationship between a tangent to a circle and a radius drawn to the point of tangency

b. Stating the relationship between a radius and a chord that it bisects

c. Determining the measures of the following types of angles given the measures of their intercepted arcs and vice versa:

(1) Central angle
(2) Inscribed angle
(3) Angle formed by a tangent and a chord drawn to the point of tangency
(4) Angle formed by two chords intersecting within the circle
(5) Angle formed by two secants
(6) Angle formed by a tangent and a secant
(7) Angle formed by two tangents

d. Stating the definitions of congruent circles

e. Stating the relationship between chords that are equidistant from the center of congruent circles

f. Solving proportions involving tangent segments, secant segments and segments of chords

g. Writing the equation of a circle given the radius and coordinates of the center and vice versa

K. Areas of Circles and Regular Polygons

20. Determines measures of circular regions by

a. Finding the measure of each interior angle of a regular polygon and their sum given the number of sides and vice versa

b. Finding the area and perimeter of a given regular polygon

c. Finding the area and circumference of a circle given its radius

d. Finding the area of a circle given its circumference and vice versa

L. Solids

21. Determines area measures of prisms, pyramids, cylinders, cones and spheres by

a. Finding the lateral surface area and the total surface area

b. Finding the area of a cross section of the solid

c. Using proportions to compare parts, cross sections and areas

d. Writing selected proofs

M. Volumes of Solids

22. Determines volume of prisms, pyramids, cylinders, cones, spheres by

a. Using proportions to compare altitudes, slant heights, radi and cross sections

b. Writing selected proofs

Algebra II (1 Credit -- Academic Mathematics)

Prerequisite: Algebra I (Required)
Geometry (Recommended)

Algebra II expands and clarifies concepts introduced in Algebra I. Emphasis is on justification of algebraic processes through basic properties and techniques in problem solving.

Upon completion of Algebra II the student should be able to

1. Understand that algebra can be viewed as a study of the structure system of numbers
2. Comprehend the function concept and its improtance in mathematics
3. Perceive the rôle of deductive reasoning in algebra
4. Appreciate the need for precision of language

ALGEBRA II (180 days)

Units

A. Functions and Relations

B. Linear Functions

C. Polynomials

D. Radicals

E. Quadratic Equations and Functions

F. Complex Numbers

G. Polynomial Functions and Equations

H. Exponents and Logarithms

I. Conic Sections

J. Systems of Equations

K. Permutations, Combinations and the Binomial Theorem

ALGEBRA II

A. Functions and Relations

1. Develops familiarity with the language for an ordered pairing of the elements of two sets by

   a. Stating the definition of a relation

   b. Writing the domain and range of a given relation from its equation and its graph

   c. Stating the definition of a function

   d. Determining if a given relation is a function

   e. Reading, writing and interpreting the following notation for functions:

      (1) $f(x) = x^2$

      (2) $f: x \rightarrow x^2$

      (3) $\{(x,y) : y = x^2\}$

   f. Graphing a given relation

B. Linear Functions

2. Expresses linear functions in forms conducive to geometric interpretations by

   a. Writing a given linear function in the form $f(x) = mx + b$ (m and b are real numbers)

   b. Describing a linear function with zero slope as a constant function

      i.e., $f(x) = b$

   c. Writing the point-slope form of the equation of a line given

      (1) a point of the line and the slope of the line
      (2) its graph

3. Applies linear functions to selected real life situations by

   a. Determining the constant of variation in a given direct variation

b. Solving selected word problems involving direct variation

C. Polynomials

4. Uses methods for writing polynomials in forms conducive to problem solving by

a. Writing the prime factorization of polynomials of the form $a^3x^3 \pm b^3y^3$

b. Solving a given factorable cubic equation

c. Showing that for all real numbers a and b,

(1) $ab > 0$ if and only if $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$

(2) $ab < 0$ if and only if $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$

d. Solving a given factorable quadratic inequality

D. Radicals

5. Recognizes whether certain properties of real numbers apply to irrational numbers by

a. Showing that the set of irrational numbers is not closed under the operations of addition, subtraction, multiplication and division

b. Finding a real number between two given real numbers

6. Illustrates that radicals are real numbers by

a. Finding all real roots of a given equation of the form $x^n = a$ (n is a positive integer)

b. Writing a given radical expression with a rational denominator

c. Adding, subtracting, multiplying and dividing given radicals

d. Solving a given equation involving at least two radicals

E. Quadratic Equations and Functions

7. Recognizes some relationships between the coefficients of a quadratic equation and its roots as an aid in problem solving by

   a. Solving a given quadratic equation by completing the square

   b. Determining the number and nature of real roots of a given quadratic equation using the discriminant

   c. Using the sum and product of the roots to

      (1) check a solution set of a given quadratic equation

      (2) write a quadratic equation

   d. Solving selected word problems

8. Uses methods of writing quadratic functions in forms conducive to geometric interpretations by

   a. Graphing a function written in the form $y = a(x - h)^2 + k$ (h and k are real numbers)

   b. Finding for a given quadratic function the

      (1) maximum or minimum value
      (2) coordinates of the vertex
      (3) equation of the axis of symmetry
      (4) solution set

   c. Sketching the curve of a given quadratic function

F. Complex Numbers

9. Uses mathematical notation to write complex numbers in a concise form by

   a. Classifying a given number as

      (1) real
      (2) imaginary
      (3) pure imaginary

   b. Evaluating a given power of i

10. Obtains logical procedures to operate on complex numbers by

    a. Writing the conjugate of a given complex number

    b. Adding, subtracting, multiplying and dividing two given complex numbers

    c. Writing a given numerical expression in the form $a + bi$ (a and b are real numbers)

    d. Solving a given equation in one variable involving complex coefficients

    e. Illustrating the field properties of complex numbers

G. Polynomial Functions and Equations

11. Relates the degree of a polynomial with the number of roots of a polynomial equation by

    a. Determining the number of roots of a given polynomial equation

    b. Solving a given polynomial equation

    c. Sketching the curve of a given cubic equation

H. Exponents and Logarithms

12. Writes concise, readable expressions using exponential notation by

    a. Multiplying and dividing powers using the properties

        (1) $x^m \cdot x^n = x^{m+n}$ (m and n are real numbers)

        (2) $\frac{x^m}{x^n} = x^{m-n}$ ($x \neq 0$, m and n are real numbers)

    b. Solving a given exponential equation

13. Recognizes that logarithms are exponents by

    a. Graphing a given exponential function and its inverse function

    b. Writing the equivalent exponential statement of a given logarithmic statement and vice versa

    c. Identifying the mantissa and the characteristic of the logarithm of a given rumber

d. Finding the approximation of given logarithms and of given antilogarithms

e. Determining the antilogarithm of a given logarithm

f. Multiplying and dividing two given positive numbers written in logarithmic form

g. Determining the logarithm of a given number

h. Solving a given logarithmic equation

i. Finding selected powers and roots of a given number using logarithms

j. Solving equations involving rational exponents using logarithms such as

(1) $5^x = 75$
(2) $x^{3/5} = 42$
(3) $10^{2x-1} = 3.175$

k. Solving selected word problems

I. Conic Sections

14. Uses terminology necessary to describe the conic sections and their characteristics by

a. Illustrating by the intersection of a plane with a conical surface of two nappes how each of the following is formed:

(1) Circle
(2) Ellipse
(3) Parabola
(4) Hyperbola
(5) Point
(6) Line
(7) Two intersecting lines

b. Identifying the following terms as they apply to each of the conic sections:

(1) Focus
(2) Directrix
(3) Vertex
(4) Axis (symmetry, major, minor, transverse, and conjugate)
(5) Center
(6) Asymptote

15. Expresses equations of conic sections in forms conducive to geometric interpretations by

    a. Writing the formula for the distance between two points given their coordinates

    b. Writing the equation of a circle given the radius and the coordinates of the center

    c. Writing an equation given in the form $Ax^2 + By^2 + Cx + Dy + E = 0$ in one of the following forms:

    (1) $(x - h)^2 + (y - k)^2 = r^2$

    (2) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

    (3) $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$

    d. Writing the equation of a conic section when given initial conditions

    e. Sketching a conic section when given its equation

    f. Sketching regions bounded by given conic sections

16. Demonstrates an understanding of the role of conic sections in selected real life situations by

    a. Determining the constant of variation when two inversely proportional numbers are given

    b. Solving problems involving combined (joint) variation

    c. Solving selected word problems involving variation

J. Systems of Equations

17. Writes sets of equations to obtain interpretable results of systems of equations by

    a. Solving a given system of three first degree equations in three variables by

        (1) addition and multiplication
        (2) substitution

    b. Solving a given system consisting of a linear and a quadratic equation and a given system consisting of two quadratic equations algebraically and graphically

    c. Solving selected word problems using systems

K. Permutations, Combinations, and the Binomial Theorem

18. Counts arrangements of objects by

    a. Finding the number of permutations of n elements, n is a natural number

    b. Finding the number of permutations of n elements taken k at a time, n and k are natural numbers and $k < n$

    c. Finding the number of combinations of n elements taken k at a time; n is a positive integer, k is a non-negative integer, and $k \leq n$

    d. Solving selected word problems involving permutations and combinations

19. Determines patterns of coefficients in a binomial expansion by

    a. Raising a given binomial to a given positive integral power

    b. Writing the expanded form of $(x + y)^n$; x and y are real numbers, n is a positive integer

    c. Constructing Pascal's triangle

    d. Writing a specified term of a given binomial expansion

Algebra II-Trigonometry (1 Credit -- Academic Mathematics)

Prerequisite: Geometry

Algebra II-Trigonometry is designed to enable the student to take a full course in Algebra II and in Trigonometry in a single course offering. The objectives for both Algebra II and Trigonometry are to be completed. Hence, the course progresses at a faster pace and with greater depth than either Algebra II or Trigonometry

ALGEBRA II-TRIGONOMETRY

Units for Algebra II

A. Functions and Relations

B. Linear Functions

C. Polynomials

D. Radicals

E. Quadratic Equations and Functions

F. Complex Numbers

G. Polynomial Equations and Functions

H. Exponents and Logarithms

I. Conic Sections

J. Systems of Equations

K. Permutations, Combinations and the Binomial Theorem

Units for Trigonometry

A. Angle, Angle Measure and Periodic Functions

B. Sine, Cosine and Tangent Functions

C. Identities

D. The Six Trigonometric and Circular Functions

E. Trigonometric and Circular Identities

F. Graphs of Circular and Related Functions

G. The Inverse Circular Functions

Trigonometry (1/2 Credit -- Academic Mathematics)

Prerequisite: Geometry and Algebra II

Trigonometry presents both a practical and theoretical viewpoint of the trigonometric and circular functions.

Upon completion of Trigonometry the student should

1. Gain knowledge of behaviors of periodic functions
2. Express in mathematical terms many of the physical laws of motion
3. Determine inaccessible distances

TRIGONOMETRY

Units

A. Angle, Angle Measure and Periodic Functions

B. Sine, Cosine and Tangent Functions

C. Identities

D. The Six Trigonometric and Circular Functions

E. Trigonometric and Circular Identities

F. Graphs of Circular and Related Functions

G. The Inverse Circular Functions

TRIGONOMETRY

A. Angle, Angle Measure, and Periodic Functions

1. Describes angles and the measures by

   a. Identifying each of the following:

      (1) Directed angles
      (2) Angles in standard position
      (3) Initial and terminal sides of an angle in standard position
      (4) Coterminal angles
      (5) Quadrantal angles
      (6) Reference angles

   b. Distinguishing between degree measure and radian measure

   c. Converting degree measure to radian measure and vice versa

2. Recognizes that certain functions are periodic by

   a. Stating the definition of periodic functions

   b. Selecting the periodic functions from a given set of functions

   c. Stating the period of a given periodic function

   d. Giving examples of periodic phenomena

B. Sine, Cosine, and Tangent Functions

3. Relates the coordinates of a point with an angle in standard position by

   a. Writing the equations of the sine ($\sin \theta = \frac{y}{r}$), cosine ($\cos \theta = \frac{x}{r}$), and tangent ($\tan \theta = \frac{y}{x}$) of a given angle in standard position where θ is given in degrees and $r = \sqrt{x^2 + y^2}$

   b. Writing the equations of the sine ($y = \sin x$), cosine ($y = \cos x$), and tangent ($y = \tan x$) of a given angle in standard position where x is given in radians and $x^2 + y^2 = 1$

c. Determining the values of sine, cosine, and tangent functions of the special reference angles $\theta \in \{0°, 30°, 45°, 60°, 90°\}$ and $x \in \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$

d. Approximating the values of the sine, cosine, and tangent functions

4. Shows that if θ is an angle with radian measure x, then the trigonometric and circular functions are equivalent

$$\text{i.e., } \left[\sin \theta = \sin \left[(\theta)\ (\tfrac{\pi}{180})\text{rad}\right] = \sin x\right]$$

C. Identities

5. Acquires a technique for problem solving by

a. Stating the following identities for the sum and difference of two angles

(1) the cosine of the sum
(2) the cosine of the difference
(3) the sine of the sum
(4) the sine of the difference

b. Deriving the following special identities (α in degrees)

(1) $\cos(-\alpha) = \cos\alpha$
(2) $\sin(-\alpha) = -\sin\alpha$
(3) $\cos(90° - \alpha) = \sin\alpha$
(4) $\sin(90° - \alpha) = \cos\alpha$
(5) $\cos(180° + \alpha) = -\cos\alpha$
(6) $\cos(180° - \alpha) = -\cos\alpha$
(7) $\sin(180° + \alpha) = -\sin\alpha$
(8) $\sin(180° - \alpha) = \sin\alpha$

(equivalent circular forms as well)

c. Becomes aware of the following identities in

(1) the tangent of the sum
(2) the tangent of the difference
(3) double angle identities
(4) half angle identities
(5) sum and product identities

6. Proves a given identity

7. Solves problems using identities

D. The Six Trigonometric and Circular Functions

8. Defines the reciprocals of the trigonometric and circular functions by

   a. Writing the equations of the cotangent ($\cot \theta = \frac{x}{y}$), secant ($\sec \theta = \frac{r}{x}$), and cosecant ($\csc \theta = \frac{r}{y}$) of a given angle in standard position where θ is given in degrees and $r = \sqrt{x^2 + y^2}$

   b. Writing the equations of the cotangent ($y = \cot x$), secant ($y = \sec x$), and cosecant ($y = \csc x$) of a given angle in standard position where x is given in radians and $x^2 + y^2 = 1$

   c. Determining the values of the cotangent, secant, and cosecant functions of the special reference angles $\theta \in \{0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ\}$ and $x \in \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$

9. Approximates values for the trigonometric and circular functions by

   a. Expressing a function for an angle greater than 90° as a function of a positive acute angle

      i.e., $\sin 240^\circ = -\sin 60^\circ$

   b. Determining the value of a given function using its reference angle

   c. Finding values for the trigonometric and circular functions by using tables

E. Trigonometric and Circular Equations

10. Demonstrates the role of trigonometry in problem solving by

   a. Solving selected word problems involving right triangles

   b. Solving equations

   c. Using the Law of Cosines and the Law of Sines to solve selected problems

F. Graphs of Circular and Related Functions

11. Illustrates the behavior of the circular and related functions by

   a. Graphing the following functions:

(1) sin x (3) tan x (5) sec x
(2) cos x (4) cot x (6) csc x

b. Determining those intervals in which the circular functions increase or decrease

c. Finding the maximum and minimum values of sin x and cos x

d. Determining the amplitude h and period k for the following functions:

(1) h sin kx (3) h tan kx (5) h sec kx
(2) h cos kx (4) h cot kx (6) h csc kx

e. Writing the equations of asymptotes of each of the following:

(1) $\{(x,y) : y = \cot x\}$
(2) $\{(x,y) : y = \tan x\}$
(3) $\{(x,y) : y = \sec x\}$
(4) $\{(x,y) : y = \csc x\}$

f. Graphing a given function and stating the composite

(1) amplitude
(2) period
(3) phase shift

g. Graphing a given function that is the sum and difference of two or more composite functions involving sine and cosine

G. The Inverse Circular Functions

12. Defines the inverse circular functions by

a. Stating the definition for the inverse of each of the circular functions (principal valued functions)

i.e., $y = \sin x \iff x = \sin^{-1} y$
$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $-1 \leq y \leq 1$

b. Stating the domain and range for the inverses of the circular functions

c. Graphing the inverses of the circular functions and stating their domain and range

13. Solves equations involving the inverse of a given circular function

Analytic Geometry (1/2 Credit -- Academic Mathematics)

Prerequisite: Trigonometry or Algebra II-Trigonometry (Required)

Analytic Geometry places emphasis on the relationships of geometric figures by means of algebraic descriptions and operations. Rectangular, polar, cylindrical and spherical coordinates as well as coordinates whose values depend upon a parameter are used to describe sets of points.

Upon completion of Analytic Geometry the student should

1. Appreciate algebra as a means of verifying geometric concepts
2. Give algebraic descriptions of lines and planes in a three-dimensional space
3. Acquire methods for classifying quadric surfaces

ANALYTIC GEOMETRY (90 days)

Units

A. Lines and Vectors in a Plane

B. The Conics

C. Translation and Rotation of Axes in a Coordinate Plane

D. Polar Coordinates

E. Lines and Planes in a Three-Dimensional Rectangular Coordinate

F. Cylindrical and Quadric Surfaces

G. Coefficient Matrix and Determinants

H. Cylindrical and Spherical Coordinates

ANALYTIC GEOMETRY

A. Lines and Vectors in a Plane

1. Uses coordinates whose values depend upon a parameter by

   a. Finding the directed distance from point A to point B on a directed line

   b. Writing the parametric equations of a line using one independent variable

   c. Finding the coordinates of the point which is at a given distance from point P_1 to point P_2 on a directed line

2. Shows a relationship that exists between two or more sets of points in a coordinate plane by

   a. Finding the distance from a line to a point

   b. Finding the tangent of the angle between two lines

   c. Writing the equations of the bisectors of the angles formed by two intersecting lines

   d. Identifying members of a family of lines which have a given geometric property

   e. Writing the equation of a family of lines which have a given geometric property

3. Describes vectors and their relationships in a plane by

   a. Determining if two given directed line segments are equivalent

   b. Adding two given vectors

   c. Expressing a vector in terms of a given unit vector

   d. Finding the magnitude of a given vector

   e. Multiplying a given vector by a constant

   f. Finding a unit vector in the direction of the given vector

   g. Finding the cosine of the angle between two nonzero vectors

   h. Finding the projection of a given vector

   i. Finding the scalar (dot) product of two given vectors

j. Determining if two given vectors are orthogonal or parallel by using the dot product

B. The Conics

4. Describes conic sections and their characteristics graphically and algebraically by

a. Identifying for each of the conics (circle, parabola, ellipse, and hyperbola) the following (as appropriate):

(1) Foci
(2) Directrices
(3) Vertices
(4) Axes (symmetry, major, minor, transverse, and conjugate)
(5) Eccentricity
(6) Center
(7) Asymptotes

b. Deriving the rectangular equations of each of the following, given sufficient information:

(1) Circle
(2) Parabola
(3) Ellipse
(4) Hyperbola

c. Sketching each of the conics

d. Writing the equation of the tangent through a given point to each of the conics

e. Identifying members of a family of circles which have a given geometric property

f. Writing the equation of a family of circles which have a given geometric property

C. Translation and Rotation of Axes in a Coordinate Plane

5. Expresses an equation in a form conducive to geometric interpretation by

a. Writing the equations of a translation of axes from origin (0,0) to origin (h,k)

b. Writing in x'y'-coordinates when given the xy-coordinates and a translation of axes each of the following:

(1) The coordinates of a point
(2) The equation of a line
(3) The equation of a conic

c. Writing in x'y'-coordinates when given the xy-coordinates and a rotation of axes each of the following:

(1) The coordinates of a point
(2) The equation of a line
(3) The equation of a conic

d. Rotating a set of axes to eliminate the xy term of a given second degree equation in two variables

D. Polar Coordinates

6. Uses a coordinate system based upon an angle and a distance by

a. Writing the polar coordinates of a point given the rectangular coordinates and vice versa

b. Transforming a given equation written in polar coordinates into a corresponding equation written in rectangular coordinates and vice versa

c. Writing in the polar coordinate system the equations for the following:

(1) Line
(2) Circle
(3) Parabola, ellipse, and hyperbola (with focus at the pole)

E. Lines and Planes in a 3-Dimensional Rectangular Coordinate System

7. Describes lines, planes, and their relationships in space by

a. Drawing and labeling the x-axis, the y-axis, the z-axis, and the origin

b. Locating a point that corresponds to a given ordered triple of numbers

c. Finding the distance between two given points

d. Finding the coordinates of the midpoint of a line segment when the coordinates of the end points are given

e. Finding the direction cosines of a line, when given the coordinates of two points of the line

f. Determining if given lines are parallel or perpendicular or neither by using direction numbers

g. Writing the parametric equations of a line using one independent variable when given sufficient information

h. Writing the equation of a line without using a parameter when given sufficient information

i. Finding the coordinates of the point of intersection of a given line and a coordinate plane

j. Writing the equation of a plane when given sufficient information

k. Determining if a given line is parallel or perpendicular to a given plane

l. Finding the cosine of the angle between two given planes

m. Determining if two given planes are parallel or perpendicular or neither

n. Finding the distance from a given point to a given plane

F. Cylindrical and Quadric Surfaces

8. Describes three-dimensional surfaces by

a. Writing the equation of a sphere

b. Writing the equation of a cylindrical surface with generators parallel to a major axis

c. Identifying the following properties of a given quadric surface:

(1) Intercepts
(2) Traces
(3) Section by a plane
(4) Symmetry

d. Identifying surfaces defined by a given equation as

(1) an ellipsoid
(2) an elliptic hyperboloid of one sheet
(3) an elliptic hyperboloid of two sheets
(4) an elliptic paraboloid
(5) a hyperbolic paraboloid
(6) an elliptic cone

G. Coefficient Matrix and Determinants

9. Uses an alternative method for solving a system of equations by

a. Stating the definition of coefficient matrix

b. Stating the definition of the determinant of a 2 by 2 matrix

c. Using determinants to solve a given system of two equations in two variables

d. Evaluating a 3×3 determinant using cofactors

e. Using determinants to solve a given system of two equations in two variables

H. Cylindrical and Spherical Coordinates

10. Relates different coordinate systems by

a. Writing the cylindrical coordinates of a point given the rectangular coordinates and vice versa

b. Writing the spherical coordinates of a point given the rectangular coordinates and vice versa

c. Writing the cylindrical coordinates of a given point given the spherical coordinates and vice versa

Analytic Geometry (3195)
Section B
September 3, 1974

Elementary Mathematical Functions (1/2 Credit -- Academic Mathematics)

Prerequisite: Trigonometry or Algebra II-Trigonometry (required)
Analytic Geometry (recommended)

Elementary Mathematical Functions places emphasis on the properties and characteristics of polynomial, trigonometric, exponential and logarithmic functions. Some basic concepts of the calculus are rigorously treated; these are limits, continuity and derivative.

The purpose of Elementary Mathematical Functions is to enable the student to

1. Develop a proficiency in representing functions graphically
2. Maintain algebraic skills and concepts
3. Become aware of some basic ideas of the calculus

ELEMENTARY MATHEMATICAL FUNCTIONS (90 days)

Units

A. Functions

B. Limits and Continuity

C. The Derivative

D. Functions (Polynomial, Trigonometric, Exponential and Logarithmic)

E. Curve Tracing and Asymptotes

F. Polynomial Equations

G. Mathematical Induction

ELEMENTARY MATHEMATICAL FUNCTIONS

A. Functions

1. Interprets functions as mappings by

   a. Distinguishing between the converse and the inverse of a function

   b. Constructing the following compositions given the definitions of functions f and g

      (1) $f\left[g(a)\right]$

      (2) $g\left[f(a)\right]$, a is a constant

      (3) $g\left[f(x)\right]$

      (4) $f\left[g(x)\right]$

   c. Identifying the domain and range of a given composite function

2. Interprets functions which involve absolute value by

   a. Stating the definition of the absolute value function

   b. Graphing a given polynomial equation involving absolute value

      e.g., $y = x^2 + |x + 2| + 6$

B. Limits and Continuity

3. Determines the existence of a limit of a function by

   a. Graphing to determine if a function has a limit as x approaches a given value

   b. Using the δ, ϵ definition of limit to show if a function has a limit as x approaches a given value

   c. Proving selected elementary limit theorems

4. Develops proficiency in determining limits by

   a. Finding limits of given functions by using the limit theorems

b. Solving problems using the definition of a limit

c. Applying the definition of continuity to prove that a given function is continuous or not continuous

C. The Derivative

5. Determines the derivative of a function by

a. Finding the slope of the tangent line to a continuous curve at a given point (assuming the tangent line is defined) by using

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \text{slope}$$

b. Illustrating that the definition of derivative is equivalent to

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

c. Using the power formula on a given polynomial function

d. Finding the derivative of functions and defined as

(1) sums
(2) quotients (including tan x)
(3) products
(4) composites

6. Illustrates a geometric interpretation of the derivative of a function by

a. Finding the slope of the tangent line to a continuous curve at a given point

b. Writing the equation of both the tangent and normal lines to a curve at a given point

7. Uses the first and second derivatives of a function to obtain and interpret the critical values of the function by

a. Stating the definition of second derivative; illustrating for a given function the relationship of the second derivative to the first derivative and to the function

b. Locating the points of a continuous function where the first derivative is zero

c. Locating the points of a continuous function where the second derivative is zero

d. Locating all maximum and minimum points and points of inflection of a given continuous function

e. Stating the relationship between maximum or minimum values of a continuous function and the first derivative

f. Stating the relationship between the maximum or minimum values of a continuous function and the second derivative

g. Applying maximum and minimum concepts to word problems such as

(1) perimeter
(2) area
(3) volume
(4) velocity

D. Functions (Polynomial, Trigonometric, Exponential and Logarithmic)

9. Describes some characteristics of a function and its graph by

a. Deciding if a given function is even or odd or neither

b. Using the following terms to describe given functions:

(1) Increasing
(2) Decreasing
(3) Nonincreasing
(4) Nondecreasing
(5) Monotonic increasing
(6) Monotonic decreasing
(7) Concave upward
(8) Concave downward

10. Obtains a pictorial representation of the real solutions of functions by

a. Predicting the effect on the graph of a given function when the coefficients are changed

b. Solving and graphing a given cubic equation

c. Stating the number of real solutions of a given polynomial function

d. Sketching the graph of a given function

E. Curve Tracing and Asymptotes

11. Describes functions graphically by

a. Stating the definition of an asymptote

b. Locating, writing the equations of, and sketching the asymptotes of a given function

c. Using the following terms as they apply to curve sketching:

(1) Domain and range
(2) Intercepts
(3) Maximum and minimum
(4) Asymptotes
(5) Plotting points
(6) Symmetry
(7) Concavity

F. Polynomial Equations

12. Solves polynomial equations by

a. Stating the Remainder Theorem; verifying the theorem for a given polynomial function

b. Stating the Factor Theorem; using that theorem to find the zeros of a given polynomial function

c. Using synthetic division to determine the factors of a given polynomial

d. Finding all rational roots of a given polynomial equation

e. Approximating irrational solutions of a given polynomial equation by using Newton's method

f. Determining the nature and number of roots of a given polynomial equation with real coefficients

g. Stating the Fundamental Theorem of Algebra

i.e., a polynomial equation of degree n has n roots

G. Mathematical Induction

13. Uses inductive reasoning as a tool in mathematical proof by

a. Comparing natural numbers with the steps of mathematical induction

b. Using mathematical induction to prove various elementary number theory formulas

Calculus AB (1 Credit -- Academic Mathematics)

Prerequisite: Analytic Geometry

Calculus AB is an Advanced Placement course which emphasizes the sound understanding of the theory of elementary functions and differential calculus of a function of one variable while introducing integration of functions of one variable.

Upon completion of Calculus AB the student should be able to

1. Develop a proficiency in representing functions graphically
2. Maintain algebraic skills and concepts
3. Become aware of some basic ideas of the calculus
4. Study motion of particles whose path cannot be defined by polynomial equations
5. Select the one element from an infinite set of elements that satisfies a given physical condition at an exact moment in time

Calculus AB (3170)
Section B
September 3, 1974

CALCULUS AB

Units

A. Functions (Polynomial, Trigonometric, Exponential and Logarithmic)

B. Limits and Continuity

C. The Derivative

D. Applications of the Derivative

E. The Integral

F. Applications of the Integral

CALCULUS AB

A. Functions (Polynomial, Trigonometric, Exponential, and Logarithmic)

1. Determines the nature of the roots of polynomial equations by

   a. Describing a polynomial function

   b. Reading and writing functional notation correctly

   c. Identifying a given function as

      (1) odd
      (2) even
      (3) neither

   d. Proving the Remainder Theorem

      i.e., the value of the polynomial function $P(x)$ for $x = a$ is the remainder when $P(x)$ is divided by $x - a$

   e. Proving the Factor Theorem

      i.e., if and only if a is a zero of $P(x)$, then $(x - a)$ is an exact divisor or factor of $P(x)$

   f. Determining factors of polynomials and roots of polynomial equations by using synthetic division

   g. Finding all rational roots of a given polynomial equation

   h. Showing that a n-th degree polynomial equation has n complex roots by using the Fundamental Theorem of Algebra (any polynomial equation has at least one complex root)

   i. Classifying the roots of a given polynomial equation as rational, irrational, or imaginary

2. Operates on functions by

   a. Determining for two given functions the

      (1) sum or difference
      (2) product
      (3) quotient

   b. Determining one of $f(x)$, $g(x)$, or $f[g(x)]$ given the other two

3. Defines the inverse of a function by

   a. Determining if an inverse function of a given function exists; if so, stating the domain and range of the inverse function

   b. Showing both geometrically and analytically, that the functions a^x and $\log_a x$ (for $a > 0$, $a \neq 1$, and $x > 0$) are inverse functions

B. Limits and Continuity

4. Determines the limit of a function by

   a. Deciding if f(x) approaches a limit as x approaches a particular value when given the graph of f(x)

   b. Developing an intuitive idea of the meaning of delta (δ) and epsilon (ϵ) by graphing

   c. Stating the δ, ϵ definition of limit

   d. Proving the following limit theorems using the δ, ϵ definition of limit:

      (1) $\lim_{x \to a} (mx + b) = ma + b.$

      (2) $\lim_{x \to a} b = b.$

      (3) $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x).$

      (4) $\lim_{x \to a} [f(x) \cdot g(x)] = \left[\lim_{x \to a} f(x)\right] \cdot \left[\lim_{x \to a} g(x)\right]$

      (5) $\lim_{x \to a} [k \cdot f(x)] = k \cdot \left[\lim_{x \to a} f(x)\right]$

5. Develops proficiency in finding limits by

   a. Using the following theorems to find limits:

      (1) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \quad \lim_{x \to a} g(x) \neq c$

      (2) $\lim_{x \to 0} \frac{k}{x}$ does not exist $\quad (k \neq 0)$

      (3) $\lim_{x \to a} f[g(x)] = f\left[\lim_{x \to a} g(x)\right]$

   b. Stating the definition of the continuity of a function at a given point and a neighborhood of the point

c. Proving if a function is continuous when a function is given

(1) graphically
(2) algebraically

C. The Derivative

6. Determines the derivative of a function by

a. Writing the slope function by using $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

b. Finding the values in the domain of a given function for which the slope function is undefined

c. Finding the slope of any nonvertical tangent line

d. Writing the definition of the derivative of a defined function

e. Finding the derivative of the power functions in the form $f(x) = ax^n$ where a is a real number and n is a rational number

f. Finding the derivatives of sin x and cos x

g. Finding the derivatives of sin (bx + c) and cos (bx + c)

h. Finding the second derivative of a given function

i. Finding the derivative of a given function, implicitly defined

j. Finding the derivative of the logarithmic function, ln x

i.e., $\log_e x$

k. Finding the derivative of the exponential functions, e^x and Ae^{kx}

l. Finding the derivative of the inverse of a given function (including arc sin x and arc tan x)

m. Finding the derivative of functions defined as

(1) sums
(2) quotients (including tan x)
(3) products
(4) composites

7. Acquires methods for representing functions graphically by

a. Finding the domain values of a given function where the first derivative is one of the following:

(1) Positive
(2) Zero
(3) Negative

b. Finding the domain values of a given function where the second derivative is one of the following:

(1) Positive
(2) Negative
(3) Zero

c. Determining maximum points, minimum points, and points of inflection of a given function by using the first and/or the second derivative

d. Determining if a given function is increasing, decreasing, nonincreasing, nondecreasing, concave upward, or concave downward by using the first and the second derivative

e. Graphing and sketching a curve using objectives c and d, included with domain, range, intercepts, asymptotes, symmetry, and point plotting (including such functions as $e^{-x}\sin x$ and $|f(x)|$)

Applications of the Derivative

8. Uses the derivative as it applies in specified situations by

a. Finding a linear approximation of $\sin x$ near $x = 0$

b. Finding polynomial approximations of $\sin x$ and $\cos x$

c. Finding an approximation of the value of e, where $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

d. Evaluating limits of functions written in an indeterminate form such as $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$, by using L'Hôpital's Rule

e. Writing the equations of both the tangent and normal lines to a curve at a given point on the curve where the derivative exists

f. Finding the slope of a curve

g. Finding the average rate of change and instantaneous rate of change of a particle whose path is defined on a plane curve

h. Locating the relative and absolute extremal values of a given function which is continuous on the closed interval $a \leq x \leq b$, and differential on the open interval $a < x < b$

i. Approximating the irrational roots of a given polynomial equation by using Newton's method

j. Finding the value(s) of x_0 using Rolle's Theorem; suppose that f is continuous for $a \leq x \leq b$ and that $f'(x)$ exists for each x between a and b; if $f(a) = f(b) = 0$, then there must be (at least) one point, call it x_0 between a and b such that $f'(x_0) = 0$

k. Finding the value(s) of x_0 using the Mean Value Theorem; suppose that f is continuous for $a \leq x \leq b$ and that $f'(x)$ exists for each x between a and b; then there is an x_0 between a and b (that is, $a < x_0 < b$) such that $f'(x_0) = \frac{f(b) - f(a)}{b - a}$.

9. Makes physical applications using the derivative by

a. Solving problems involving related rates of change

b. Finding the velocity and acceleration of a particle moving along a given plane curve

c. Solving extremal value word problems for

(1) perimeter
(2) area
(3) volume

E. The Integral

10. Demonstrates the relationships between the derivative and area under a curve to the integral by

a. Stating the definition of the definite integral and listing the properties

b. Determining the indefinite integral of a given function

c. Evaluating the definite integral of a given function by using the Fundamental Theorem of Integral Calculus

d. Approximating the area under a given curve using

(1) upper and lower sums
(2) the trapezoidal rule

e. Determining the integral of a given function by the method of substitution

F. Applications of the Integral

11. Uses the integral as it applies in specified situations by

a. Finding the average (mean) value of a given function

b. Finding the area bounded by two given plane curves

c. Finding volumes of solids with regular cross sections, including solids of revolution

d. Finding polynomial approximations of sin x, cos x, e^x, e^{-x}, and ln (1 + x) by using integration and inequality

e. Interpreting ln x as area under the graph of $y = x^{-1}$

f. Finding distance and velocity from acceleration with given initial conditions

12. Applies the integral to the solutions of differential equations by

a. Finding general solutions of $y^{(n)} = 0$ (nth derivative identically zero)

b. Finding the general solutions of $y'' = -k^2y$; applying these solutions to solve simple harmonic motion problems

c. Finding general solutions of the differential equations: $y' = ky$ and $y = kx^{-1}$; applying these solutions to solve problems involving growth and decay

Calculus BC (1 Credit -- Academic Mathematics)

Prerequisite: Analytic Geometry and Elementary Mathematical Functions

Calculus BC is an Advanced Placement course which emphasizes theoretical aspects of the calculus of functions of a single variable and includes integrals, infinite series, and differential equations.

Upon completion of Calculus BC the student should begin to

1. Recognize that certain sums of infinitesimal quantities having an infinite number of terms converge to a real number

2. Study motion of particles whose path cannot be defined by polynomial equations

3. Select the one element from an infinite set of elements that satisfies a given physical condition at an exact moment in time

Calculus BC (3171)
Section B
September 3, 1974

CALCULUS BC (180 days)

Units

A. Fundamental Concepts

B. Derivative

C. Applications of Derivative

D. The Integral

E. Transcendental Functions

F. Techniques of Integration

G. Applications of Definite Integral

H. Geometry in the Plane

I. Sequences and Series

J. Elementary Differential Equations

CALCULUS BC

A. Fundamental Concepts

1. Determines limits and continuity of a function of one variable by

   a. Finding the least upper bound of a bounded subset of the real numbers

   b. Finding the greatest lower bound of a bounded subset of the real numbers

   c. Reading and writing functional notation correctly for functions in one independent variable

   d. Proving by using the epsilon, delta definition the following:

      (1) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \quad \lim_{x \to a} g(x) \neq 0$

      (2) $\lim_{x \to a} f\left[g(x)\right] = f\left[\lim_{x \to a} g(x)\right]$

      (3) If $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists and if $\lim_{x \to a} g(x) = 0$,

      then $\lim_{x \to a} f(x) = 0$.

   e. Finding limits using the following theorems:

      (1) $\lim_{x \to a} (mx + b) = ma + b$

      (2) $\lim_{x \to a} b = b$

      (3) $\lim_{x \to a} \left[f(x) \pm g(x)\right] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

      (4) $\lim_{x \to a} \left[f(x) \cdot g(x)\right] = \left[\lim_{x \to a} f(x)\right] \cdot \left[\lim_{x \to a} g(x)\right]$

      (5) $\lim_{x \to a} \left[k \cdot f(x)\right] = k \left[\lim_{x \to a} f(x)\right]$

      (6) All theorems listed in objective 1d above

   f. Proving by using the properties of continuous functions if a function is continuous when the function is defined either graphically or algebraically

B. The Derivative

2. Illustrates the relationships between slope, limit and continuity as they apply to differentiability by

   a. Using the definition of derivative to find the derivative of polynomial and trigonometric functions

   b. Illustrating the existence of continuous functions that are not differentiable

   c. Showing that all differentiable functions are continuous

   d. Finding the value(s) of x_0 using Rolle's Theorem; suppose that f is continuous for $a \leq x \leq b$ and that $f'(x)$ exists for each x between a and b. If $f(a) = f(b) = 0$, then there must be (at least) one point, call it x_0, between a and b such that $f'(x_0) = 0$

   e. Finding the value(s) of x_0 using the Mean Value Theorem; suppose that f is continuous for $a \leq x \leq b$

3. Uses efficient methods for determining the derivative by

   a. Determining the derivatives of functions written as

      (1) Sums
      (2) Products
      (3) Quotients
      (4) Composites

   b. Determining the derivative of functions implicitly defined

C. Applications of the Derivative

4. Uses the derivative as it applies in given situations by

   a. Writing equations of tangent and normal lines to a given curve

   b. Sketching curves using

      (1) domain and range
      (2) intercepts
      (3) maximum and minimum points
      (4) asymptotes
      (5) point plotting
      (6) symmetry
      (7) concavity
      (8) convexity

c. Locating relative and absolute extremal values of a function which is continuous on the closed interval $a \leq x \leq b$ and differentiable on the open interval $a < x < b$

d. Finding velocity and acceleration of a particle whose path is defined along a straight line

e. Solving problems involving related rates of change

f. Using the principal part of the increment to find a linear approximation of a function

g. Using L'Hôpital's rule for finding limits when applicable

D. The Integral

5. Determines that the area under a curve relates to the integral of a function by

a. Stating the definition of the definite integral and listing the properties

b. Evaluating the definite integral of a given function and determining the indefinite integral of a given function

c. Showing when limits of sums are equivalent to definite integrals

d. Approximating area under a curve using

(1) upper and lower sums
(2) the Trapezoidal Rule
(3) Simpson's Rule

e. Evaluating $\int_a^b f(x)dx$ where $f(x)$ is continuous on $a \leq x \leq b$ and $\int f(x)dx = F(x) + c$ (Fundamental Theorem of Integral Calculus)

f. Showing geometrically the relationship between natural logarithms and the area under a curve

E. Transcendental Functions

6. Uses the derivative and the integral as they apply to trigonometric, exponential and logarithmic functions by

a. Showing that e^x and $\ln x$ are inverse functions

b. Finding the derivative of $\ln x$ and the derivative of e^x

c. Determining the derivative of $\ln u$ where u is a positive differentiable function of x

d. Finding general solutions of the differential equations $y' = ky$ and $y' = kx^{-1}$; applying these solutions to problems involving growth and decay

e. Deriving the derivatives of trigonometric functions

f. Determining indefinite integrals of trigonometric functions by considering the integral and the derivative as inverse operations

g. Finding the general solutions of $y'' = -k^2y$; applying these to find solutions of simple harmonic motion problems

h. Determining the derivative of a^x and the derivative of $\log_a x$ (for $a > 0$, $a \neq 1$, and $x > 0$)

i. Graphing a^x and $\log_a x$ (for $a > 0$, $a \neq 1$, and $x > 0$)

j. Showing that a^x and $\log_a x$ are inverse functions (for $a > 0$, $a \neq 1$, and $x > 0$)

k. Determining the derivatives of arc sin u and arc tan u, where $u = f(x)$

F. Techniques of Integration

7. Writes sets of equivalent expressions to obtain an expression that can be efficiently integrated by the methods of simple substitution by

a. Using simple substitution (using identities and change of variable)

b. Using integration by parts

c. Using trigonometric substitutions

d. Using partial fractions

G. Applications of the Definite Integral

8. Uses the integral as it applies in given situations by

a. Finding the average (mean) value of a function

b. Finding the area bounded by two plane curves

c. Finding the area bounded by curves expressed by polar coordinate equations

d. Finding volumes of solids with regular cross sections, including solids of revolution

e. Finding the length of an arc

f. Finding surface areas of solids with regular cross sections, including solids of revolution

g. Solving work problems

h. Evaluating improper integrals

H. Geometry in the Plane

9. Develops a proficiency in operating on vectors by

a. Expressing a vector in terms of unit vectors

b. Adding vectors

c. Multiplying a vector by a constant

d. Finding the magnitude of a vector

e. Finding a unit vector in the direction of a given vector

f. Finding the cosine of the angle between two nonzero vectors

g. Finding the projection of a vector on a vector

h. Finding the scalar (dot) product of two vectors

i. Showing vectors are orthogonal or parallel by use of the dot product

j. Defining a plane curve by two parametric equations written with respect to the same independent variable

k. Finding the velocity vector and acceleration vector of a particle whose path is defined along a plane curve

l. Finding the radius of curvature

I. Sequences and Series

10. Recognizes the meaning of convergence by

a. Determining if a sequence of real numbers converges

b. Showing that bounded monotonic sequences converge

c. Determining if series of real numbers converge by using the

(1) comparison test
(2) ratio test
(3) integral test

d. Showing that absolute convergence implies convergence

e. Showing that convergence does not necessarily imply absolute convergence

f. Determining if a sequence of functions converge

g. Determining if a series of functions converge

h. Determining if a power series converges; if so, finding the radius of convergence

i. Finding polynomial approximations of transcendental functions using Taylor's series

J. Elementary Differential Equations

11. Recognizes that families of curves which are described in terms of slope relate to specific physical problems by

a. Finding general solutions of first order differential equations whose variables are separable

b. Solving first order linear differential equations

c. Solving second order linear differential equations with constant coefficients of the following types:

(1) Homogeneous
(2) Nonhomogeneous (elementary examples)

d. Making applications with differential equations given initial conditions

Computer Science I (1/2 Credit -- Academic Mathematics)

Prerequisite: Algebra I and recommendation of mathematics teacher or teachers

Computer Science I provides opportunities for students to become familiar with the fundamental structure of a computer system and to discover the uses of computers applicable to many disciplines. Skills in writing and running programs on a computer are developed through an individualized approach that allows the student to work with both mathematical and nonmathematical problems.

Because Computer Science I and Computer Science II are both part of a continuous course, a scope and sequence chart is provided following the Computer Science II description.

Computer Science II (1 Credit -- Academic Mathematics)

Prerequisite: Algebra II or Algebra and Trigonometry or concurrent enrollment in Algebra, Trigonometry, or Algebra and Trigonometry

Computer Science II is an introductory two-semester course on the fundamental topics and issues of the computer field. The topics in Computer Science I are presented in Computer Science II in more depth and detail.

The additional semester provides individualized training in a second programming language and an introduction to assembly and machine language. The latest techniques in problem solving using computers are presented so that a student can continue study at an advanced level in computer science, mathematics or other related fields.

Programs are to be written in a chosen area of interest limited only by the student's ingenuity and mathematical knowledge, and by the capability of the computer and access provided.

Computer Science I (3180)
Computer Science II (3181)
Section B
September 3, 1974

Computer Science I (Units 1 and 2)
Computer Science II (Units 1-4)

CONTENT

UNIT I (45 days)

Development of Computer Systems	1. Becomes aware of the early development of computer systems.
Algorithms	1. Becomes aware of the meaning of algorithms. 2. Reads and writes the symbols for illustrating an algorithm with a flowchart. 3. Derives numerical algorithms and uses flowchart symbols to illustrate the algorithm for a given problem.
Computer Access	1. Demonstrates the logging on procedures for accessing a computer and the logging off procedures for terminating the access when the access is via a terminal. 2. Demonstrates with a given program the operational functions of a terminal.
Computer Communication	1. Gains a readiness for using a programming language. 2. Gains a readiness for implementing a coded program on a computer system.
Input/Output	
Storage	1. Determines the fundamental structure of storage through the study of other number systems. 2. States the meaning of a code and distinguishes between number systems and codes to further determine the fundamental structure of storage.
Arithmetical and Logical Structures	
Control	1. Demonstrates the provision for control in terms of the programming and the control unit of a computer system.
Fortran	
Assembly and Machine Language	
Input/Output Channel Controllers	
Operating Systems	

UNIT 2 (45 days)

2. Becomes aware of the current development of computer systems.

4. Derives iterative and non-numerical algorithms and uses flowchart symbols to illustrate the algorithm for given problems.

3. Writes and runs programs for given problems.

1. Discusses the input and output of information in a computer system using correct data processing terms.

1. Designs a structure for determining the arithmetic and logical capabilities of a computer through the use of Boolean algebra.
2. Shows inutitively that through addition of real numbers the other three fundamental operations can be derived.

2. Discusses the provision for control of the computer system by the human operator.

UNIT 3 (45 days)

5. Derives converging algorithms for given problems.

2. States how a card is read by the input component of a computer system and how information is printed using the output component.
3. Identifies the hardware components for storage and states their function in terms of number systems and codes.

3. Subtracts binary numbers by adding their complements.
4. Shows that subtraction in a computer is accomplished by first finding the complement of a number and then using the adder circuit.

1. Demonstrates the arithmetic capabilities of FORTRAN.
2. Demonstrates the control capabilities of FORTRAN.
3. Demonstrates the capabilities of writing and running a program in FORTRAN.

1. States the major functions of an operating system.

UNIT 4 (45 days)

6. Defines problems that can be solved by using a computer.

3. Describes other methods of input and output in a computer system with other storage mediums such as magnetic tape and disc.

4. Describes the methods used for storing and transferring information in computer systems.

5. Shows that multiplication and division in a computer are accomplished by shifting registers and using respectively addition and subtraction circuitry.

3. Discusses the impact the computer has made in our society.

4. Writes and runs programs using FORTRAN.
5. Illustrates the advanced capabilities of FORTRAN.

1. Recognizes that a machine language identifies the capabilities of a given computer system.
2. Recognizes that an assembly language provides a programmer a means for convenient and efficient use of the capabilities of a given computer system.
3. Writes and runs programs using an assembly language.
1. Explains how channel controllers can activate, monitor and terminate input and output devices efficiently.
2. Recognizes that a job control language is necessary to communicate with the operating system.
3. Shows the need for an operating system by recognizing various computer structures and techniques.

COMPUTER SCIENCE I and COMPUTER SCIENCE II

UNIT 1

DEVELOPMENT OF COMPUTER SYSTEMS

1. Becomes aware of the early development of computer systems by

   a. Discussing the capability of man to operate on numbers with mechanical devices prior to the 20th century
   b. Discussing the capability of man to operate on numbers with electro-mechanical devices prior to World War II

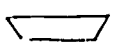
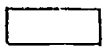
ALGORITHMS

1. Becomes aware of the meaning of algorithms by

   a. Stating and illustrating an algorithm (step-by-step process) to accomplish any given task
   b. Recognizing that a partial algorithm is necessary to define the step-by-step process in accomplishing a specific part of a generalized task
   c. Showing that some algorithms involving extensive numerical and information processing procedures require a computer to accomplish the task efficiently (computer-oriented algorithms) by

      (1) Recognizing that a numerical algorithm is used to manipulate sets of numbers so that a requested solution to a problem can be obtained
      (2) Recognizing that a non-numerical algorithm is used to manipulate sets of information so that a requested arrangement and analysis of the information can be obtained
      (3) Recognizing that some problems are solved by an iterative algorithm which is a repeating procedure with a specified beginning and a controlled termination
      (4) Recognizing that some problems are solved by a converging algorithm which requires defining a convergence test and acceptable criteria for the problem solution

2. Reads and writes the symbols for illustrating an algorithm with a flowchart such as

   a. Initiation and termination
   b. Input and output 
   c. Flow direction (arrows)
   d. Connector
   e. Processing 
   f. Decision

3. Derives numerical algorithms and uses flowchart symbols to illustrate the algorithm with a flowchart such as

   a. Finding the greatest of 3 numbers; of 4; of 5; and finally of 'n'
   b. Arranging (sorts) a set of numbers given in a random sequence so that the set of numbers is in increasing (ascending) order
   c. Finding the square roots of a given number
   d. Determining whether a given number is prime or composite
   e. Finding the greatest common factor and the least common multiple of a given pair of integers

COMPUTER ACCESS

1. Demonstrates the logging on procedures for accessing a computer and the logging off procedures for terminating the access when the access is via a terminal.

2. Demonstrates with a given program the operational functions of a terminal by

   a. Entering a program
   b. Running a program
   c. Modifying a program
   d. Terminating a program
   e. Storing a program

COMPUTER COMMUNICATION

1. Gains a readiness for using a programming language by

   a. Identifying the kind of translator (assembler, compiler, or interpreter) of a computer system and states that the function of a translator is to change a source program (written by a programmer in source code) to an object program (executed by the computer in machine code)
   b. Stating and identifying the purposes for various programming languages such as the following:

      (1) ASSEMBLER (machine oriented)
      (2) BASIC (introductory and instructional oriented)
      (3) FORTRAN (science and mathematics oriented)
      (4) COBOL (business oriented)

   c. Stating for a given programming language the rules for coding a statement consisting of one or more of the following:

      (1) Constants
      (2) Variables
      (3) Operators
      (4) Expressions

   d. Coding statements in a given language that provides the following functions:
      (1) Acceptance of input and generation of output
      (2) Assignment of values to variables

(3) Control of program through statement transfer such as
(a) Looping (iterative process)
(b) Branching (conditional--based on a comparison test or unconditional--no test)
(c) Accommodating subroutines (a set of statements that is a program in itself and can be included or excluded from another program)

2. Gains a readiness for implementing a coded program on a computer system by

a. Interacting with a given program to determine and correct the following errors (debugging):
(1) Source diagnostics (source code, error messages)
(2) Object time errors (errors exceeding machine capabilities)
(3) Program logic errors (algorithm and/or statement usage errors)

b. Interacting with the computer to manipulate program and data files using commands to perform the functions, such as the following:
(1) Editing (correcting and changing program)
(2) Storing (saving programs)
(3) Listing (printing out requested programs)
(4) Running (executing object code)
(5) Linking (combining two or more programs)

STORAGE

1. Determines the fundamental structure of storage through the study of other number systems by

a. Writing the generalized form ($d_nB^n + d_{n-1}B^{n-1} + \ldots + d_1B^1 + d_oB^o$) to express a number with a given base

b. Converting a number in a given base to a number in a specified different base. Specifically:

From	To
(1) Decimal	Binary Pental Octal Hexidecimal
(2) Binary Pental Octal Hexidecimal	Decimal
(3) Binary	Octal (without the use of decimal numbers) Hexidecimal (without the use of decimal numbers)
(4) Octal Hexidecimal	Binary (without the use of decimal numbers)

c. Performing the four fundamental operations on binary numbers

d. Stating that the binary number system is used to represent information in computers and lists reasons for the use by considering two-way devices

2. States the meaning of a code and distinguishes between number systems and codes to further determine the fundamental structure of storage by
   a. Coding and translating information on punched cards using the Hollerith code
   b. Translating the alphanumeric character set (the 10 digits of the decimal system and 26 letters of the alphabet to the BCD (Binary Coded Decimal) code and vice versa
   c. Interpreting the full set of 128 USASCII (USA Standard Code for Information Interchange) code combinations by using a chart

CONTROL

1. Demonstrates the provision for control in terms of the programming and the control unit of a computer system by completing the following:

   a. Listing and interpreting the meaning of the steps to be used in writing, entering, and running a computer program
      (1) Problem definition
      (2) Flowcharting
      (3) Coding
      (4) Compiling
      (5) Testing (debugging)
   b. Naming the two parts of an instruction
      (1) Operation code (to indicate which operation the computer is to perform)
      (2) Operand (to direct and modify the specifications of the operation)
   c. Recognizing the control unit as the main unit of a computer for accomplishing the following:
      (1) Loading the program
      (2) Fetching the instruction
      (3) Executing the instruction

UNIT 2

DEVELOPMENT OF COMPUTER SYSTEMS

2. Becomes aware of the current development of computer systems by

   a. Discussing the capabilities and limitations of the following unit record machines
      (1) Key punch
      (2) Sorter
      (3) Collator
      (4) Reproducing punch
      (5) Interpreter
      (6) Accounting machine
   b. Identifying the purposes for which calculators, stored program digital computers, and analog computers are used

c. Evaluating the capabilities and limitations of first, second, and third generation computers
d. Identifying the function of certain hardware components for a digital computer system such as
   (1) Card-read punch
   (2) Magnetic tape unit
   (3) Magnetic disc unit
   (4) Central processing unit
   (5) Printer

ALGORITHMS

4. Derives iterative and non-numerical algorithms and uses flowchart symbols to illustrate the algorithm for given problems such as

   a. Determining if a given positive integer is even or odd and using the result to develop a repeating series of 3 numbers (4,2,1) by continuously applying the following rules (iterative algorithm)
      (1) If the result is even, divide the result by 2.
      (2) If the result is odd, multiply the result by 3 and add 1
   b. Selecting positive integer values of k, l, and m, and finding the number of and the sum of all positive integers greater than k and less than l divisible by m (iterative algorithm)
   c. Writing a program to calculate the compound interest on $2,000 at 4% compounded quarterly for nine years and having the program print the principal and year number at the end of each year (iterative algorithm)
   d. Writing a program to print all sets of three integers less than 20 so that they can be the sides of a right triangle (iterative algorithm)
   e. Providing the capability of taking a given string of characters and changing any part of the string (a contiguous substring of any length) to another substring of any length. For example, given the string COMMPUTOR, the substring MPUTO is to be replaced by PUTE, and the original string is to be changed to COMPUTER (non-numerical algorithm)

COMPUTER COMMUNICATION

3. Writes and runs programs for given problems.

INPUT/OUTPUT

1. Discusses the input and output of information in a computer system using correct data processing terms such as
   a. Source data
   b. Editing information
      (1) Validity
      (2) Accuracy
      (3) Rearrangement

c. Files
   (1) Records
      (a) Fields
      (b) Control (key) fields
   (2) Master
   (3) Transaction
   (4) Sequential
   (5) Random Access
d. Manipulating Files
   (1) Updating
   (2) Sorting
   (3) Merging

ARITHMETICAL AND LOGICAL STRUCTURES

1. Designs through the use of Boolean algebra a structure for determining the arithmetic and logical capabilities of a computer by

   a. Stating and writing the symbols for the operators--And, Or, and Not--and illustrates their meaning by use of circuitry designs and/or Venn diagrams
   b. Verifying the following identities, postulates, and theorems of Boolean algebra by use of a truth table
      (1) Identities
         (a) $A + 0 = A$
         (b) $A \cdot 0 = 0$
         (c) $A + 1 = 1$
         (d) $A \cdot 1 = A$
         (e) $A + A = A$
         (f) $A \cdot A = A$
         (g) $A + A' = 1$
         (h) $A \cdot A' = 0$
         (i) $(A')' = A$

      (2) Postulates
         (a) Commutative: $A + B = B + A$; $A \cdot B = B \cdot A$
         (b) Associative: $A + (B + C) = (A + B) + C$; $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
         (c) Distributive: $A \cdot (B + C) = A \cdot B + A \cdot C$; $A + (B \cdot C) = (A + B) \cdot (A + C)$

      (3) Theorems
         (a) Absorption: $A \cdot (A + B) = A$; $A + A \cdot B = A$
         (b) DeMorgan's: $(A \cdot B)' = A' + B'$; $(A + B)' = A' \cdot B'$
         (c) Derived: $A + A' \cdot B = A + B$

   c. Simplifying Boolean expressions using Boolean identities, postulates, and theorems
   d. Stating how binary addition is accomplished by the half-adder and full-adder circuit
   e. Constructing the truth table and deriving the logical function for the half-adder and for the full-adder using Boolean algebra

2. Shows intuitively that through addition of real numbers the other three fundamental operations can be derived.

CONTROL

2. Discusses the provision for control of the computer system by the human operator for actions such as

   a. Starting and stopping the computer system
   b. Selecting jobs and placing priorities
   c. Interrupting any process performed by the computer and making required changes

UNIT 3

ALGORITHMS

5. Derives converging algorithms for given problems such as

   a. Refining the Euclidean algorithm for finding the G.C.D. for two given numbers by

      (1) Assign A to the variable L and B to the variable S
      (2) To show that the G.C.D. of A and B converges to L, test for S is zero.
      (3) If S is zero, the G.C.D. of A and B is L.
      (4) If S is not zero, then the reiterative process for determining Q (the quotient of L and S); R (the remainder for Q) is continued until S is 0

   b. Locating the real zeros for a given polynomial by use of the Location Principle which in graphical terms is stated as follows

      If point (x_1, P_{x_1}) and point (x_2, P_{x_2}) are on opposite sides of the x-axis, then the graph must cross the x-axis between $(x_1, 0)$ and $(x_2, 0)$ (converging algorithm)

INPUT/OUTPUT

2. States how a card is read by the input component of a computer system and how information is printed using the ouput component.

STORAGE

3. Identifies the hardware components for storage and states their function in terms of number systems and codes by

   a. Showing how (by a drawing) magnetic core storage holds and transmits information
   b. Listing the types of main memory (internal storage) and input/output storage (external storage) currently in use or in development. States the advantages and disadvantages of each type in terms of cost, access time and capacity

ARITHMETICAL AND LOGICAL STRUCTURES

3. Subtracts binary numbers by adding their complements.

4. Shows that subtraction in a computer is accomplished by first finding the complement of a number and then using the adder circuit.

FORTRAN

1. Demonstrates the arithmetic capabilities of FORTRAN by

   a. Using the general rules for coding integer (fixed point) and real (floating point) for preparing numeric data as:

      (1) Input for a program
      (2) Constants in a program

   b. Recognizing the codes for and the order of the five basic arithmetic operations provided in FORTRAN
   c. Writing and determining the meaning of arithmetic expressions using constants, variables, and operators
   d. Recognizing that there are limitations of arithmetic such as

      (1) Magnitude
      (2) Precision
      (3) Associativity

   e. Determining what functions are provided and states the code for each function
   f. Determining the value of an expression (integer and/or real) by applying arithmetic assignment statement rules
   g. Recognizing the structure of the READ, WRITE and FORMAT statements

2. Demonstrates the control capabilities of FORTRAN by

   a. Using the GO TO statement to transfer control unconditionally
   b. Using the arithmetic IF, the computed GO TO, and the logical IF to transfer control conditionally
   c. Recognizing the PAUSE statement to interrupt a program, the STOP statement to terminate a program, and the END statement to terminate the compilation

3. Demonstrates the capabilities of writing and running a program in FORTRAN by

   a. Selecting and coding statements for a given problem using a coding sheet
   b. Interpreting diagnostic messages of the compiler, and making required code changes

c. Using type statements and providing dimension requirements (when necessary) to code the following kinds of variables

(1) Double precision
(2) Complex
(3) Subscripted
(4) Logical

d. Making required source code error adjustments as indicated during the execution phase of the compiler

OPERATING SYSTEMS

1. States the major functions of an operating system such as

a. Scheduling loading, initiating, and supervising the execution of programs by priority criteria
b. Allocating system resources such as storage and input/output
c. Initiating and controlling input/output operations
d. Keeping records of system usage (accounting)
e. Protecting memory
f. Providing utility and library program services
g. Handling interruptions
h. Coordinating communication between the human operator and the computer

Unit 4

ALGORITHMS

6. Defines problems that can be solved by using a computer.

INPUT/OUTPUT

3. States and describes other methods of input and output in a computer system with other storage mediums such as magnetic tape and disc.

STORAGE

4. Describes the methods used for storing and transferring information in computer systems by

a. Identifying the use of registers and stating why they are required in computers
b. Illustrating (with a drawing) parallel and serial modes of transferring data in a computer
c. Stating how parity checking is used to determine the correctness of information transfer

ARITHMETICAL AND LOGICAL STRUCTURES

5. Shows that multiplication and division in a computer are accomplished by shifting registers and using respectively addition and subtraction circuitry.

CONTROL

3. Discusses the impact the computer has made in our society by

   a. Identifying the advantages and/or disadvantages of using the computer in
      (1) Education
      (2) Government
      (3) Business
      (4) Law

   b. Identifying the extent of control and/or influence a computer has on

      (1) Information on individuals (privacy)
      (2) Production
      (3) Technical applications

         (a) Obsolescence
         (b) Advancement

FORTRAN

4. Writes and runs programs using FORTRAN.

5. Illustrates the advanced capabilities of FORTRAN by using programming statements such as

   a. The DO statement
   b. The implied DO in Input/Output statements
   c. The FORMAT statement
   d. Statement functions
   e. FUNCTION andSUBROUTINE subprograms
   f. EQUIVALENCE, COMMON, DATA, and EXTERNAL statements

ASSEMBLY AND MACHINE LANGUAGE (GENERALIZED)

1. Recognizes that a machine language identifies the capabilities of a given computer system by

   a. Illustrating that variable-length instruction and/or fixed-length instruction formats direct a requested operation for the computer to perform
   b. Stating the instruction format and valid instruction length(s) for a given computer system

c. Illustrating the significance of determining the exact time for an instruction to go through its cycles when the time is a relevant programming factor to be considered in a computer system
d. Describing the addressing technique for storage of a given computer system
e. Describing how the control unit distinguishes between operation codes and address codes
f. Stating that by using instructions and/or control switches the operator can manipulate the hardware circuitry for computer operations such as

(1) Running programs
(2) Displaying the content of storage and/or registers
(3) Special features, e.g., modifying storage or register content, executing requested instructions

g. Stating the operation of the registers during instruction fetching and execution

2. Recognizes that an assembly language provides the programmer a means for convenient and efficient use of the capabilities of a given computer system by

a. Identifying the significant features using the following terms:

(1) Mnemonic operation codes
(2) Symbolic operands
(3) Literal operands
(4) Storage definition statements
(5) Assembler control statements
(6) Macro instructions

b. Coding a set of instructions using a coding sheet to illustrate the functions of the assembler in converting symbolic codes to machine codes
c. Distinguishing source statement categories by using examples for the following:

(1) Declarative
(2) Imperative
(3) Control
(4) Macro

d. Illustrating with examples various types of address operands such as

(1) Symbolic
(2) Actual
(3) Literal

3. Writes and runs programs using an assembly language by

   a. Preparing input and output formats
   b. Selecting and coding instructions for specified input/output requirements
   c. Interpreting diagnostic messages of the assembler and making required code changes
   d. Correlating a program to the hardware components of a computer by using programming techniques such as

      (1) Iterative processes
      (2) Address and program modification
      (3) Indexing with information tables
      (4) Subroutine linkage

   e. Analyzing and interpreting machine codes from program listings and core dumps to determine program errors (debugging)
   f. Specifying program requirements using the following criteria:

      (1) Problem definition
      (2) Program testing (test data versus actural data usability)
      (3) Program operational procedures
      (4) Program maintenance (adaptability of program to changes)
      (5) Documentation (for future use)

INPUT/OUTPUT CHANNEL CONTROLLERS

1. Explains how channel controllers can activate, monitor and terminate input and output devices efficiently by

   a. Illustrating the functions of channel controllers by correctly using the following terms:

      (1) Device activation
      (2) Cycle stealing
      (3) Protection
      (4) Rate comparison
      (5) Interruption
      (6) Buffer storage (command, subcommand, and data registers)

   b. Stating that I/O channel controllers allow multiple program capabilities by optimizing the use of the central processing unit and I/O devices

OPERATING SYSTEMS

2. Recognizes that a job control language is necessary to communicate with the operating system.

3. Shows the need for an operating system by recognizing various computer structures and techniques such as

   a. Batch processing
   b. Multi-processing
   c. Multi-programming
   d. Time-sharing

Probability and Statistics (1/2 Credit -- Academic Mathematics)

Prerequisite: Algebra II or Algebra II-Trigonometry

Probability and Statistics is a course in a career oriented curriculum applicable to many disciplines. The course provides opportunities for students to conduct scientific experiments and to become familiar with the nature of statistics. Emphasis is placed on problem solving using rules informally developed.

Upon completion of Probability and Statistics the student should be able to

1. Effectively collect and analyze data
2. Predict what will happen under circumstances that include both known elements and chance
3. Make applications of mathematics to both the physical and nonphysical sciences
4. Make decisions based on statistical methods

PROBABILITY AND STATISTICS (90 days)

Units

A. Probability

B. Description of Measurements

C. Probability Distributions

D. Statistical Inference

E. Statistical Inference Involving Two Populations (Supplementary)

F. Linear Regression and Correlation (Supplementary)

PROBABILITY AND STATISTICS

A. Probability

1. Obtains organizational procedures for analyzing data by

   a. Recognizing data which can be classified as empirical

   b. Conducting a random sample experiment by describing and selecting a sample from a population

   c. Stating the definitions for simple and compound events; listing examples of each kind of event

2. Makes reasonable conjectures from purely chance phenomena by

   a. Expressing the probability of an event as a fraction

   b. Stating the definition for the probability of an event

   c. Computing the probability of an event, P (E), in a finite sample by summing the probabilities of the sample points in the event of interest

3. Acquires an efficient method for counting arrangements of objects by

   a. Identifying permutations and combinations as counting events in a sample

   b. Using the following permutation and combination theorems in problem solving:

      (1) $P_r^n = n(n - 1)\ (n - 2) \ldots (n - r + 1)$

      (2) $C_r^n = \frac{n!}{n!(n - r)!}$

4. Describes relationships between two or more events by

   a. Classifying two or more events as one or more of the following:

      (1) Complementary
      (2) Conditional
      (3) Dependent
      (4) Independent
      (5) Mutually exclusive

b. Stating the definition for the conditional probability of two events A and B

i.e., $P(B/A) = P(AB)/P(A)$ or $P(A/B) = P(AB)/P(B)$

c. Proving that two events are independent if either $P(A/B) = P(A)$ or $P(B/A) = P(B)$

5. Predicts the outcome of a combination of events given the results of at least one of those events by

a. Computing the conditional probability of two events that are either dependent or independent

b. Using the following laws of probability in calculating the probability of a compound event:

(1) Additive: $P(A+B) = P(A) + P(B) - P(AB)$

(2) Multiplicative: $P(AB) = P(A)\ P(B/A) = P(B)\ P(A/B)$

B. Description of Measurements

6. Expresses sums in a concise form by

a. Using correctly the symbols $\sum_{i}^{n} x_i$ to denote summation

b. Using for summations the following theorems:

(1) $\sum_{i}^{n} c = nc$

(2) $\sum_{i}^{n} cx_i = c \sum_{i}^{n} x_i$ (c = a constant)

(3) $\sum_{i}^{n} (x_i + y_i + z_i) = \sum_{i}^{n} x_i + \sum_{i}^{n} y_i + \sum_{i}^{n} z_i$

7. Reports the information contained in data by

a. Preparing and using histograms in describing given sets of measurements

b. Using the terms "population" and "sample" correctly and recognizing that a sample is a subset of a given population

c. Finding for a given experiment the following measures of central tendency:

(1) Mean [$\bar{x}$ (sample), μ (population)]

(2) Median

(3) Mode

d. Finding for a given experiment the following measures of variability:

(1) Range

(2) Variance [s'^2 (sample variance), s^2 (sample variance with $(n - 1)$ degrees of freedom), σ^2 (population)]

(3) Standard deviation [σ (population), S (sample)]

e. Computing the mean $\bar{x}$ and the sample variance s'^2, when given a population sample

f. Using the sample measures $\bar{x}$ and s^2 to estimate the population measures μ and σ

g. Stating and applying the Empirical Rule (as the rule relates to Tchebysheff's Theorem)

C. Probability Distributions

8. Applies the past success of an event to future trials by

a. Stating the four requirements for a binomial experiment

b. Deriving the binomial probability distribution

i.e., $P(x) = C_x^n p^x q^{n-x}$

c. Computing the binomial probability distribution for a given experiment

d. Computing the mean and standard deviation for the binomial variable in a given binomial experiment

9. Minimizes the chance of error in accepting or rejecting a given hypothesis by

a. Constructing graphically the operating characteristic curve for a given sampling plan

b. Defining for a given experiment the following:

(1) Null hypothesis (H_o)
(2) Acceptance region
(3) Rejection region
(4) Type I errors (α) and Type II errors (β)

D. Statistical Inference

10. Draws useful inferences from samples applied to the normal distribution by

    a. Stating the Central Limit Theorem and applying the theorem to a given experiment

    b. Constructing the normal probability distribution curve and interpreting its tabulated values as areas lying within a specified number of standard deviations of the mean

    c. Approximating the binomial distribution by use of the normal distribution and computing for a given experiment the following:

        (1) Values of z_i (the standardized normal variable)
        (2) Probabilities for x_i (the random variable)
        (3) Probability of a Type I error and location of the rejection region.
        (4) Probability of a Type II error and location of the acceptance region

    d. Distinguishing between a confidence interval and a confidence coefficient

11. Demonstrates successful sampling procedures to evaluate a hypothesis by

    a. Stating the hypothesis to be tested

    b. Selecting the sample

    c. Stating the alternative hypothesis

    d. Evaluating Type I or Type II errors

E. Statistical Inference Involving Two Populations (Supplementary)

12. Demonstrates successful sampling procedures to evaluate a hypothesis involving two populations by completing each of the following:

    a. Testing a hypothesis concerning μ (the mean) of two populations by use of the student's distribution

    b. Computing and making inferences from small samples concerning the difference between two means

    c. Identifying the Chi-square probability distribution and the F-distribution; interpreting tabulated values for each kind of distribution

    d. Drawing inferences from the varianges of two given populations by using the Chi-square test and the F-test

F. Linear Regression and Correlation (Supplementary)

13. Uses a straight line as a predictor of the outcome of an experiment by

a. Constructing for a given experiment a scatter diagram

b. Obtaining for a given experiment a prediction equation by use of the scatter diagram and the line of "best fit" method

c. Using the least squares method to find the "best" fitting line for a given set of points

d. Computing and analyzing σ^2 (the variance of the random error of the deviation from the regression line) by using the data for a given experiment

e. Using the regression equation $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ to

(1) Test $H_o : \beta_1 = 0$ against $H_a : \beta_1 \neq 0$

(2) Estimate the slope β_1

(3) Compute the expected value of y, given x, $[E(y/x)]$

f. Computing the coefficient of correlation between y and x for a given line

g. Defining and using a prediction equation for a given experiment

Senior Mathematics (1/2 Credit -- Academic Mathematics)

Prerequisite: Algebra II, and senior students

Senior Mathematics is a thematic approach to the study of student selected topics. The course aims at satisfying student needs by offering opportunities to concentrate on selected themes

The purpose of Senior Mathematics is to enable the student to

1. Explore areas of mathematics relevant to the student
2. Acquire some knowledge of selected areas of mathematics

The class selects four four-week units from the following list of units:

1. Consumer Mathematics for Seniors
2. Vectors
3. Probability
4. Matrices
5. History of Mathematics
6. Progressions, Series, and Binomial Theorem

The remaining two weeks of the course may be used to expand upon one or more of the selected topics or to introduce any one of the following topics:

7. Number Theory
8. Mathematical Induction
9. Statistics (after Probability if selected)
10. Transformational Geometry
11. Elementary Point Set Topology

SENIOR MATHEMATICS

Units

A. Consumer Mathematics for Seniors

B. Vectors

C. Probability

D. Matrices

E. History of Mathematics

F. Progressions, Series and the Binomial Theorem

SENIOR MATHEMATICS

A. Consumer Mathematics for Seniors

1. Employment

   a. How to choose a job

   b. Procedure for getting a job

   c. Consideration of salary, fringe benefits and withholding taxes.

2. Financial Institutions

   a. Governing of our monetary system by the Federal Reserve Board

   b. Identification of kinds of and services rendered by financial institutions

   c. Computation of interest based on various kinds of rates

3. Transportaion

   a. Consideration of initial costs of purchasing a mode of transportation and the running and maintenance costs thereafter

   b. Comparison of costs and advantages of various kinds of public transportation

4. Housing

   a. Comparison of renting versus purcha ing costs

   b. Consideration of operating and maintenance costs

5. Insurance

   a. Reasons for buying insurance

   b. Consideration of various kinds of insurance policies and the premiums and benefits thereof

   c. Comparisons of group versus individual plans

   d. Computation of social security rates as a payrolle deduction and listing of future benefits

6. Income Tax

   a. Completion of federal tax form 1040 and state tax form 760

   b. Explanation of role of taxation in the economy

7. Investment

   a. Recognition of the relationship of return rates and the national economy

   b. Consideration of various kinds of investments

8. Consumer Wisdom

   a. The making of a personal budget

   b. Discrimination in the selection of articles to be purchased

   c. Comparison of cash versus credit buying

   d. Recognition of the advantages of record keeping

B. Vectors

1. Terms

   a. Directed line segment

   b. Scalar

   c. Vector

   d. Unit vectors

2. Operations

   a. Product of a scalar and a vector

   b. Addition

      (1) Polygon Method
      (2) Parallelogram Method
      (3) Component Method

   c. Subtraction (as inverse addition)

   d. Dot (inner) product

3. Properties

   a. Closure

   b. Commutativity

   c. Associativity

   d. Existence of an identity vector

   e. Existence of inverses

   f. Scalar distribution over addition

4. Applications

   a. Proof of certain selected geometric theorems such as

      (1) Distance Formula
      (2) Midpoint Formula
      (3) Parallelism of Vectors
      (4) Perpendicularity of Vectors

   b. Resolution of forces in physics

C. Probability

1. Terms and Laws

   a. Empirical data

   b. Sample population

   c. Simple and compound events

   d. Probability of an event

   e. Permutations and combinations

   f. Kinds of events

      (1) Complementary
      (2) Conditional
      (3) Dependent
      (4) Independent
      (5) Mutually exclusive

   g. Conditional probability of two events

   h. Laws of probability

      (1) Additive: $P(A+B) = P(A) + P(B) - P(AB)$
      (2) Multiplicative: $P(AB) = P(A)P(B/A) = P(B)P(A/B)$

2. Distributions

a. Random variables

(1) Discrete
(2) Continuous

b. Mathematical expectations of a given discrete random variable

c. Binomial probability distribution

D. Matrices

1. Equality and Order of Matrices

2. Operations

a. Addition

b. Inverses

c. Multiplication

3. Determinants Associated with Special Matrices

a. Cramer's Rule

b. Reduction of a 4 x 4 matrix

c. Systems of linear equations

E. History of Mathematics

1. The Evolution of Mathematics Through Two Basic Sources

a. Geometry

b. Algebra

2. A Brief Chronological and Biographical History of Mathematical Growth

a. Replacement of nomads for farmers

(1) Seasons
(2) Boundaries

b. Ancient oriental (near and far east)

(1) Rudiments of arithmetic and algebra
(2) Forlklore and superstition
(3) "Rhind Papyrus" (1550 B.C.)

c. Greek mathematics (1000 B.C.-250 A.D.)

(1) "Why?" as well as "How?"
(2) Geometry
(3) Developers, compilers and innovators

a. Thales
b. Pythagoras
c. Plato
d. Euclid
e. Archimedes
f. Appollanius
g. Ptolemy
h. Diophantus

d. Middle East and India

(1) Aryabata
(2) Brahmagupta
(3) Omar Khayyam
(4) Al-Khwarisimi
(5) Alhazenge

e. Western Europe (Middle Ages)

(1) Fibonacci
(2) Regiomantus
(3) Tartaglia
(4) Cardano
(5) Viete
(6) Napier

f. 17th Century

(1) Galileo
(2) Descartes
(3) Pascal
(4) Newton
(5) Leibniz

g. 18th Century

(1) Bernoulli brothers
(2) Euler
(3) La Grange

h. 19th Century

(1) Gauss
(2) Galois
(3) Abel
(4) Hamilton
(5) Riemann
(6) Weierstrass
(7) Kronecher, Cantor, Dedekind
(8) Lohachevski
(9) Cayley and Sylvester
(10) Hilbert

i. 20th Century

(1) Einstein
(2) Gödel
(3) Von Braun
(4) Whitehead
(5) Russell

F. Progressions, Series and the Binomial Theorem

1. Terms and Sums of Sequences

a. Arithmetic sequences

(1) Common difference
(2) Arithmetic mean
(3) Sum of terms

b. Geometric sequences

(1) Common ratio
(2) Geometric mean
(3) nth term
(4) Sum of n terms

2. Limits and Sums of Geometric Series with the Absolute Value of the Ratio Less Than 1 ($|r| < 1$)

a. Limit

b. Sum

3. The Binomial Expansion

a. Pascal's triangle

b. The rth term ($r \leq n$)

c. General procedure for expansion

Consumer Mathematics (1 Credit -- Academic Mathematics)

Prerequisite: A grade 11 or grade 12 student with 1 credit in mathematics
A grade 10 student with 1 credit in geometry

Consumerism is becoming a most important part of today's society. Topics which consider personal money management help the student become a judicious consumer.

Upon completion of Consumer Mathematics the student should be able to

1. Learn about the operation and procedures of business in American economy
2. Develop arithmetical skills useful to him as both a worker and a consumer
3. Handle financial responsibilities wisely
4. Expose all possible alternatives and opportunities for consumer consideration
5. Think of himself in the market place
6. Increase relevance of schooling

CONSUMER MATHEMATICS (180 days)

Units

A. Basic Expenditures

1. Food
2. Clothing
3. Housing
4. Transportation

B. Income

C. Budgeting

D. Population Trends and the Job Market

E. Taxation

F. Credit

G. Loans

H. Banking

I. Insurance

J. Investment

K. Advertising and Fraudulent Practice

CONSUMER MATHEMATICS

A. Basic Expenditures

Food

1. "Buys" nutritious food wisely by

   a. Using properties (associative and distributive) to strengthen mental arithmetic, for efficient shopping

   b. Comparing the market prices of several types of a selected kind of food by using unit pricing (price per unit of liquid or dry measure)

   c. Comparing cost and food value for a selected kind of food by making a chart such as the one shown below

Form of Food	Cost	Food Value
Frozen		
Fresh		
Canned		
Prepackaged		
Home-frozen		
Home-canned		
Home-grown		
Organic		

   d. Comparing for a selected kind of food the cost of name brand items to the cost of the store brand

   e. Comparing for a selected kind of food the cost of buying in quantity to the cost of buying in smaller units.

Clothing

2. "Buys" quality clothing by

   a. Comparing the costs for a selected kind of clothing by making a chart such as the one shown below:

Kind of Clothing	Cost
Ready-made	
Custom-made	
Self-made	

   b. Making a list of the various kinds of fabrics For each kind in the list, writing the quality in terms of comfort, durability, attractiveness, and cost

   c. Stating the advantages and disadvantages of buying a selected item at a cost discount or special sale price

   d. Reading and comprehending the written guarantee on selected wear-dated items

   e. Listing the advantages of buying washable items

Housing

3. Recognizes the advantage of considering money factors before making an investment in a house by

   a. Recognizing the importance of location in selecting a house

   i.e. (1) Considering the proximity of housing to shopping centers, places of employment, recreation, churches, schools, hospitals, public transportation, and parking facilities

   (2) Considering the availability and quality of the water supply, sewage and garbage disposal, and utility services

   (3) Interpreting and evaluating local zoning laws and considering how often these laws are changed

   (4) Evaluating community pollution levels such as

   (a) Air
   (b) Water
   (c) Noise

   (5) Evaluating community's crime rate and social makeup

   b. Determining the costs of buying a house by completing the following objectives

i.e. (1) Listing the qualifications a person must have to obtain each of the following kinds of loans: conventional, FHA-VA, FHA, VA, and subsidized

(2) Making a chart to compare the costs for a loan of a given amount such as the chart shown below:

Kind of Loan	Down Payment	Interest Rate	Number of Payments	Amount of Payments	Finance Charges
Conventional					
FHA-VA					
FHA					
VA					

(3) Comparing the costs and the coverages of various types of insurance for a house of a given assessed value by making a chart

(4) Determining what takes place at a closing and the closing costs

(5) Determining the taxes on a subsidized and unsubsidized house of a given assessed value

c. Showing the difference between renting and buying

i.e. (1) Comparing costs over a two-year period of owning a house of a given amount to renting a comparable space by considering items for owning such as

(a) Maintenance and upkeep
(b) Appreciation
(c) Depreciation
(d) Assessment rates
(e) Community taxes
(f) Frequency of assessments
(g) Income taxes
(h) Down payments
(i) Finance charges
(j) Closing costs
(k) Insurance
(l) Utility costs
(m) Equity
(n) Foreclosure

(2) Considering items for renting such as

(a) Maintenance
(b) Utility cost
(c) Insurance
(d) Rent payment and other fees

(3) Reading and interpreting landlord and tenant regulations and lease agreements for renting a given house or apartment

d. Making a report on the cost, advantages and disadvantages of various types of housing available such as tents, caves, trailers, communes, apartments, condominiums, boats, hotels, and houses

e. Considering other expenditures for housing by completing the following objectives

(1) Estimating the costs of furnishings such as rugs, furniture, appliances, and accessories

(2) Estimating maintenance costs for a ten-year period for items such as interior and exterior repair and decorating, walkways, driveways, and grounds

(3) Comparing the cost of contracted to do-it-yourself for a given maintenance problem and losts the advantages and disadvantages of each method

Transportation

4. Recognizes the importance of using judgment in selecting a mode of transportation by completing the following objectives:

a. Identifying the cost for owning a selected new mode of transportation and a selected used mode by making a chart such as

Mode of Transportation	Selling Price	Insurance	Taxes	Maintenance	Depreciation	Operating Costs

b. Identifying and comparing the costs of traveling a given distance using several modes of transportation by making a chart such as:

Mode	Number of Miles To Be Traveled	Cost per Mile	Total Cost

c. Comparing by computing the costs of buying a selected mode of transporation using the discounts available through local organizations (such as United Buyers Association) to the costs for buying the mode using installment loans from various lending institutions

d. Recognizing hazards which may be involved in buying a given mode of transportation such as

(1) Faulty workmanship

(2) Expensive maintenance and running costs

(3) Inefficient service garages and stations

(4) Lack of warranty

(5) Number of factory recalls

e. Analyzing types and comparing liability and collision insurance from a selected insurance company for a selected motor vehicle

f. Comparing the cost and benefits offered by various insurance companies for a given mode of transportation by making a chart

B. Income

5. Recognizes that the difference between gross and net income plays a major role in determining the outgo of personal money sources by completing the following

a. Computing for a hypothetical worker or for himself the total income from sources such as

(1) Salaries or wages

(2) Tips

(3) Fringe benefits

(4) Interests

(5) Dividends

(6) Gifts

b. Computing withholdings and deductions from a given gross income by considering items such as

(1) Federal and state income tax

(2) Social security tax

(3) Insurance

(4) Professional dues

(5) Retirement benefits

c. Comparing by subtraction and by ratio the amount of gross income (actual salary) with the amount of net income (take-home pay)

C. Budgeting

6. Recognizes that record keeping and planning for the outgo of personal money can make efficient handling of money possible by completing the following

a. Estimating expenditures for an individual or a family budget for items such as

(1) Housing
(2) Clothing
(3) Food
(4) Transportation
(5) Medical expense
(6) Recreation
(7) Insurances
(8) Savings and investments

b. Constructing sample budget by determining priorities (needs versus desires)

D. Population Trends and the Job Market

7. Recognizes the advantages in considering the availability of and the qualifications for certain kinds of jobs when considering personal occupations by completing the following

a. Making a research report on past, present, and future population trends according to age groups, geographical location, education, and training

b. Showing by a graph depicting clusters of population density that the mobility of the population affects the job market and vice versa

c. Using current research to determine estimates of future trends of the changing job market based upon factors such as

(1) Population trends
(2) Minority groups
(3) Technology
(4) Gross national product

d. Recognizing the advantages of vocational guidance in providing accurate and current analysis of occupational trends, tests of abilities and interests, and assistance in job placement

E. Taxation

8. Computes the amount of taxation on a given income by completing the following

a. Completing a sample federal and state income tax by using each of the following forms

(1) Standard deduction
(2) Short form
(3) Long form
(4) Itemized deductions

b. Computing the amount of property tax on a given assessed value at the given tax rates

c. Computing the amount of community taxes in a given community

d. Computing the amount of utility tax on a given utility

e. Computing the tax on a given amount for items such as food, clothing and entertainment

f. Determining the cost of the penalties for late payment of a given tax

g. Showing by a graph that personal and corporate income taxes are a large percentage of federal revenue sources

F. Credit

9. Recognizes the advantages and disadvantages of buying on credit by completing the following

a. Identifying four means for installment buying and computing the total cost for buying a selected item for a given time period using each of the following means

(1) Remaining balance

(2) Average daily balance

(3) Sum of average daily balance

(40 Payment of last in-first out and first in-last out

b. Computing the credit costs for purchasing a selected item on a revolving charge account for a given period of time

c. Identifying the various types and the fees involved for various kinds of credit card services such as single use, multiple use, and travel and entertainment

d. Interpreting a written statement of credit card charges and regulations and computing the payments for a selected item for a given period of time

e. Computing a sample application for a credit card

f. Computing the borrowing and purchasing costs of a selected item at a typical pawnshop

G. Loans

10. Recognizes the various places and procedures for obtaining a loan by completing the following

a. Recognizing the different types of financial institutions and the services they offer

(1) Commercial banks and trust companies

(2) Federal savings and loans

(3) Mutual savings and loans

(4) Finance corporations

(5) Credit unions

(6) Loan sharks

b. Reading and using tables of simple and compound interest to determine the amount required to repay a loan of a given amount

c. Recognizing the requirements necessary to obtain a loan such as income, assets, collateral, and character

d. Making a list to show the advantages and disadvantages of the following kinds of loans

(1) Secured loans

(2) Demand notes

(3) Life insurance loans

H. Banking

11. Recognizes that correct use of bank services can be a means for efficient money management by completing the following

a. Identifying various banking services by stating the purpose of services such as

(1) Checking accounts

(2) Savings accounts

(3) Loans

(4) Custodial accounts

(5) Safe-deposit boxes

(6) Traveler's checks

(7) Certified and cashier checks

b. Writing and endorsing a sample check

c. Keeping a sample checking account and making a sample of the corresponding bank statement

d. Naming some of the legal aspects involved with a checking account such as forging checks, stopping payments, and countersigning

e. Simulating the procedures for handling checks which have been written with insufficient funds for clearing

f. Computing by using a formula various ways of compounding interest such as annual, semi-annual, and monthly.

g. Reading a table to compute interest compounded daily and by minutes

h. Computing the amount of interest earned for a given amount with various types of savings such as regular savings, time deposits (CD), and U.S. Treasury bonds

I. Insurance

12. Recognizes that insurance is usually voluntarily bought to protect persons and property by completing the following:

a. Listing reasons for buying life insurance such as

(1) Support of family in event of death

(2) Income for old age

(3) Availability of a loan

(4) Investments for future use

b. Distinguishing between term life insurance and cash value life insurance

c. Listing the purposes for the following four kinds of life insurance

(1) Term

(2) Straight

(3) Limited pay

(4) Endowment

d. Making a chart to determine and compare the costs and benefits of each of the four kinds of life insurance for a $1,000 policy

e. Distinguishing between mutual and stock insurance companies and between participating and non-participating companies

f. Interpreting policies for and the limitations of the following kinds of health insurance

(1) Hospital
(2) Surgical
(3) Medical
(4) Major medical
(5) Disability

J. Investments

13. Recognizes that investments which increase personal income depend upon personal decision making and upon the economy of the nation by completing the following:

a. Identifying and listing the advantages and disadvantages of the following investments

(1) Stocks
(2) Bonds
(3) Mutual funds

b. Computing sample yields from a given investment

c. Distinguishing between common and preferred stock by computing a sample yield from each

d. Distinguishing between different types of bonds such as government savings, corporate, and municipal by computing a sample yield from each

e. Interpreting daily results of a selected stock and bond transaction from a newspaper table

f. Computing the net profit or loss from a selected stock transaction

g. Identifying other types of investment such as real estate, annuities, and profit sharing

h. Computing sample investments through a given monthly investment plan or a given investment club

K. Advertising and Fraudulent Practices

14. Recognizes that advertising and fraudulent practices can lead to financial loss by completing the following:

a. Identifying hazards of borrowing and lending such as

(1) Hidden clauses
(2) Excessive interest rates
(3) Inadequate financial and character references

b. Comparing advertising statements for a given product (through newspapers, magazines, television, radio, road signs, flyers, telephone, and mail media) with the actual cost and productivity of the product

c. Identifying some of the hazards in purchasing marked-down items such as

(1) Bent or swollen cans
(2) Discolored meat
(3) Stale or old food
(4) Damaged fabrics
(5) Unsafe cars
(6) Outmoded merchandise

d. Recognizing the value of checking open-dating when purchasing perishable items

e. Identifying some fraudulent practices used by sellers such as

(1) Used car salesmen
(2) Door-to-door salesmen
(3) Temporary companies
(4) Contractors